# Geomechanical models in DuMu$^x$

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# Balance equations (el2p model)

- Mass and momentum balance for the fluid

$$\frac{\partial(\phi_{\mathrm{eff}}\,\rho_\alpha S_\alpha)}{\partial t} - \mathrm{div}\left\{\rho_\alpha \frac{k_{\mathrm{r}\alpha}}{\mu_\alpha}\,\mathbf{K}_{\mathrm{eff}}\,(\mathrm{grad}\,p_\alpha - \rho_\alpha\,\mathbf{g}) + \phi_{\mathrm{eff}}\,S_\alpha\,\rho_\alpha \frac{\partial \mathbf{u}}{\partial t}\right\} = q_\alpha$$

$$\phi_{\mathrm{eff}} = \frac{\phi_0 - \mathrm{div}\ \mathbf{u}}{1 - \mathrm{div}\ \mathbf{u}}$$ (Han and Dusseault, 2003)

$$k_{\mathrm{eff}} = k_0 \exp\left[22.2\,(\phi_{\mathrm{eff}}/\phi_0 - 1)\right]$$ (Rutqvist and Tsang, 2002)

Discretised with Box method

- Momentum balance for the solid

$$\mathrm{div}(\Delta\boldsymbol{\sigma}' + \Delta p_{\mathrm{eff}}\,\mathbf{I}) - \phi\,S_{\mathrm{n}}\,(\rho_{\mathrm{n}} - \rho_{\mathrm{w}})\,\mathbf{g} = 0$$

Discretised with standard Galerkin FE scheme

Darcis (2013)

# Constitutive Equations

- Linear elastic behavior (Hooke‘s Law)

  $$\boldsymbol{\sigma} = \lambda \mathrm{tr}[\boldsymbol{\epsilon}] + 2\mu\boldsymbol{\epsilon}$$

- Strain derived from the displacement

  $$\boldsymbol{\epsilon} = \frac{1}{2}(\mathrm{grad}\,\mathbf{u} + \mathrm{grad}^{\mathrm{T}}\mathbf{u})$$

- Sum of fluid saturations equals 1

  $$\Sigma_\alpha S_\alpha = 1$$

- Pressures are connected via the capillary pressure (calculated from the Brooks-Corey relation)

  $$p_n = p_w + p_c(S_\alpha)$$

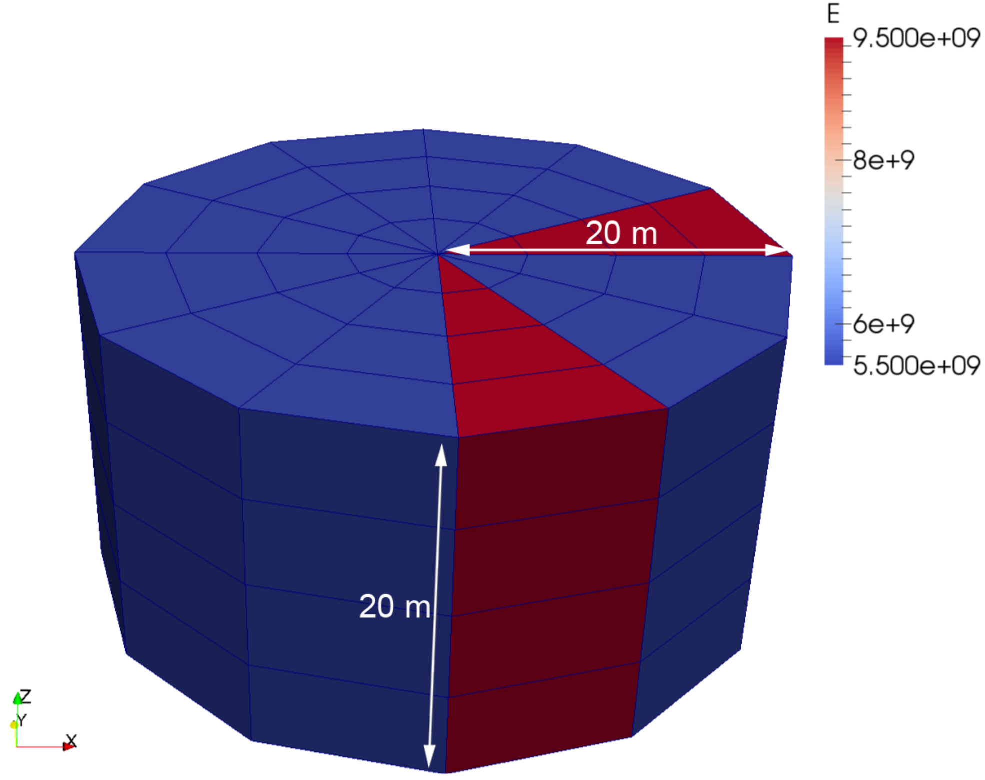
# Radial injection scenario

- Neumann no-flow for all faces except the borehole where 40 kg/s (0.032 m³/s) of $CO_2$ are injected

- Dirichlet: $u_x$, $u_y$, $u_z = 0$ for all lateral faces and the bottom

  Neumann: $\Delta\boldsymbol{\sigma} = 0$ for the top

- $\phi = 0.2$, $\boldsymbol{K} = 1 \cdot 10^{-14}$ m²/s



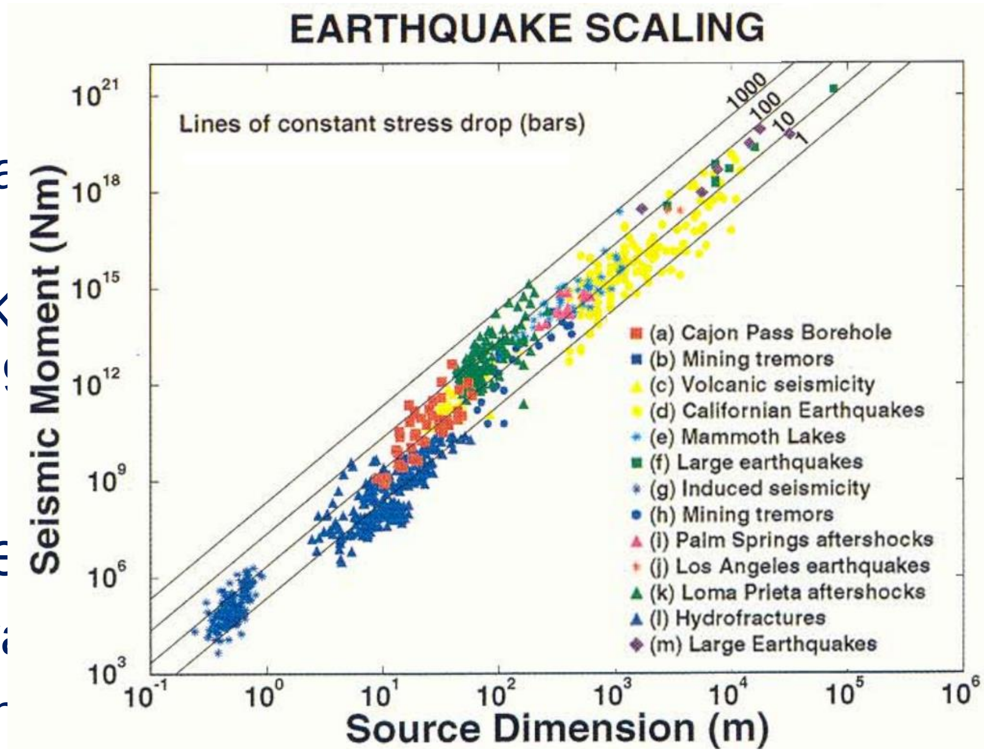
# Shear failure

- Failure evaluation with Mohr-Coulomb:

$$p_{\mathrm{crit}} = \sigma_{\mathrm{m}} - \frac{|\tau_{\max}| - S_0 \cos\varphi}{\sin\varphi}$$

- Stress drop:
  - Difference in stress before a
  - Confined to 1-10 MPa over Thatcher & Hanks (1973), K Abercrombie and Leary (19

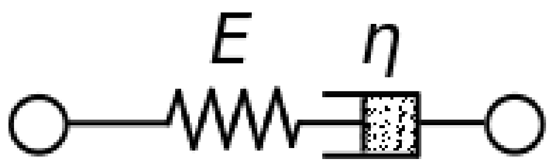
- Energy balance during se
  - Elastic energy -> seismic wa
  - Seismic event ≙ Dissipatior



Abercrombie and Leary (1993))

# Phenomenological equivalent to failure

- Maxwell material
  - Hookean spring, Newtonian dashpot

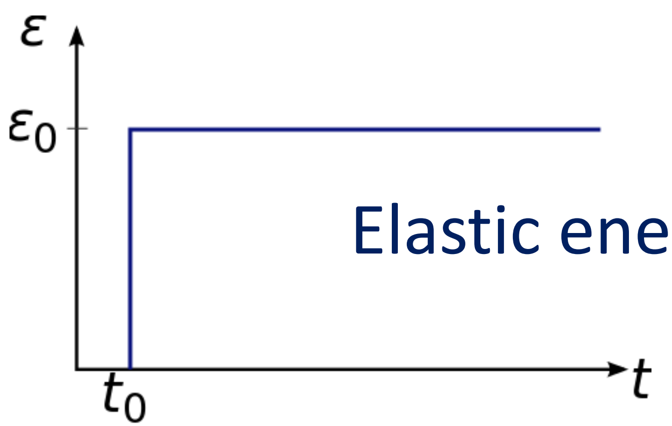


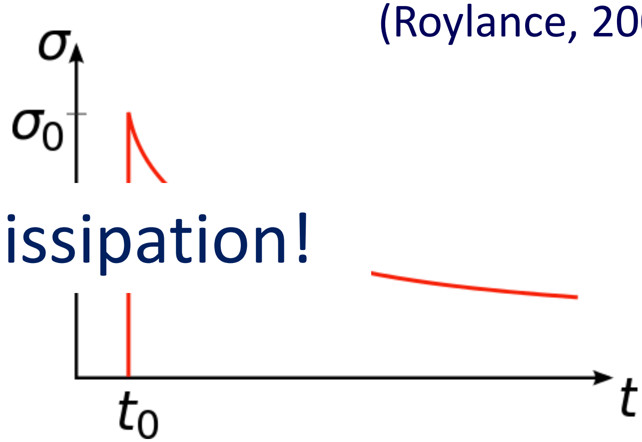

$$\frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} = \dot{\epsilon}$$ (Roylance, 2001)

  - Constant strain $\varepsilon_0$

$$\sigma(t) = \sigma_0 \exp(-t\, E/\eta)$$

(Roylance, 2001)



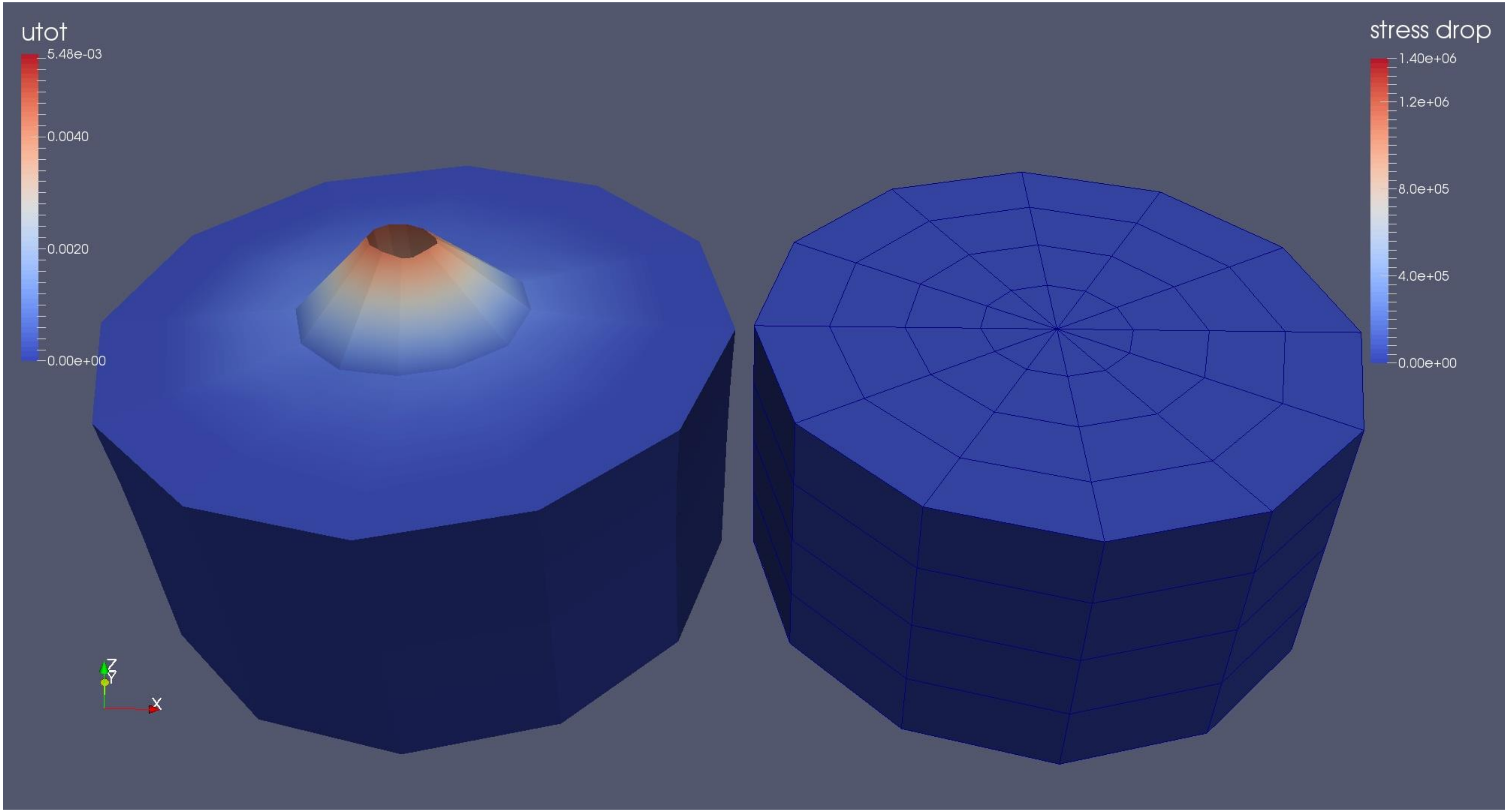




Elastic energy dissipation!

# Implementation details

- Post-timestep routine
  - Calculation of the principal stresses (eigenvalues of the stress tensor
  - Evaluation of failure criterion
  - Tag failing elements
  - If failure has happened in the previous timestep:
    determine irreversible displacement and store stress drop

- Pre-timestep routine
  - Update whether elastic ($E$, $v$) or viscoelastic ($E(\eta)$, $B$) properties are used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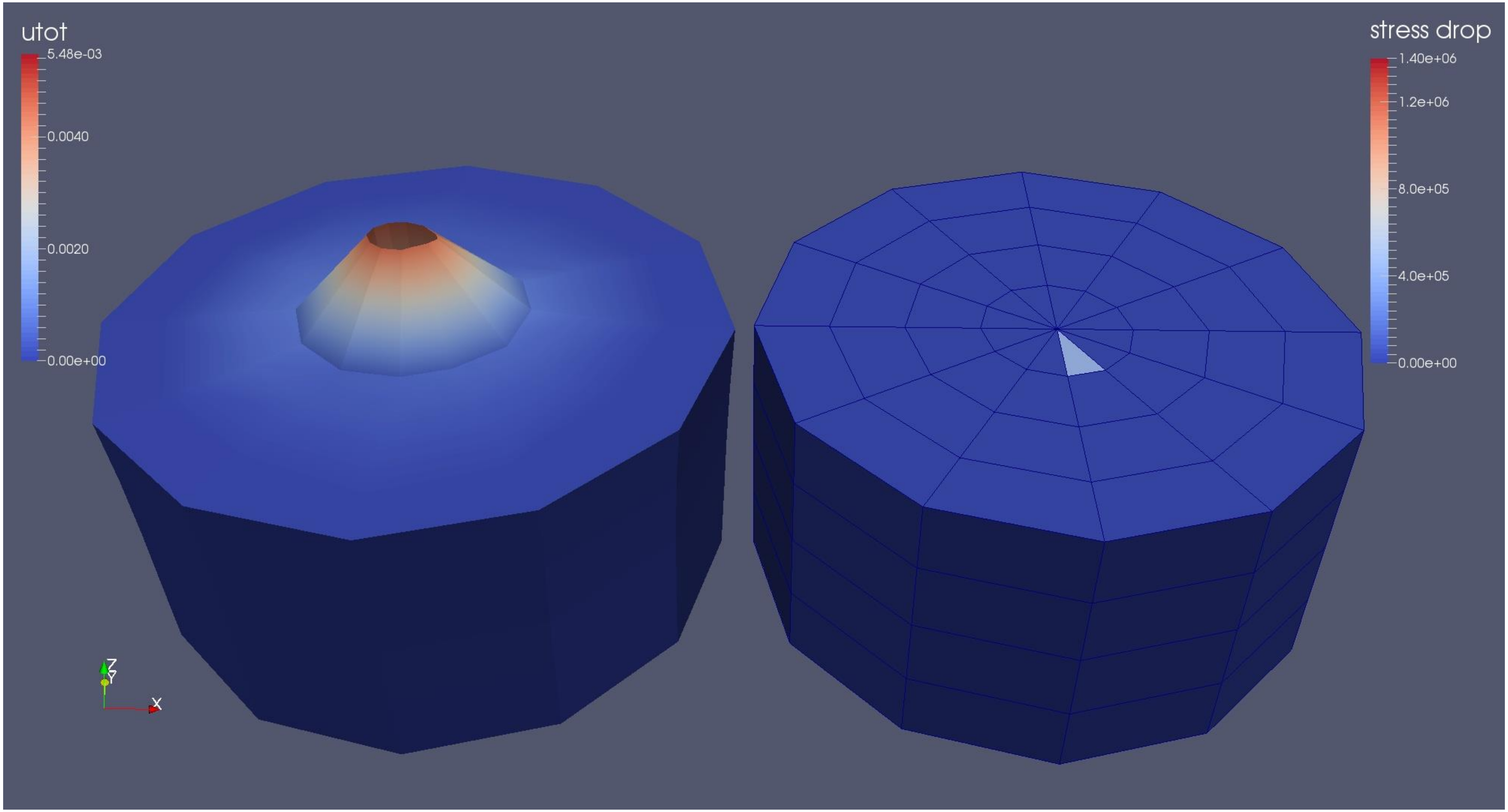
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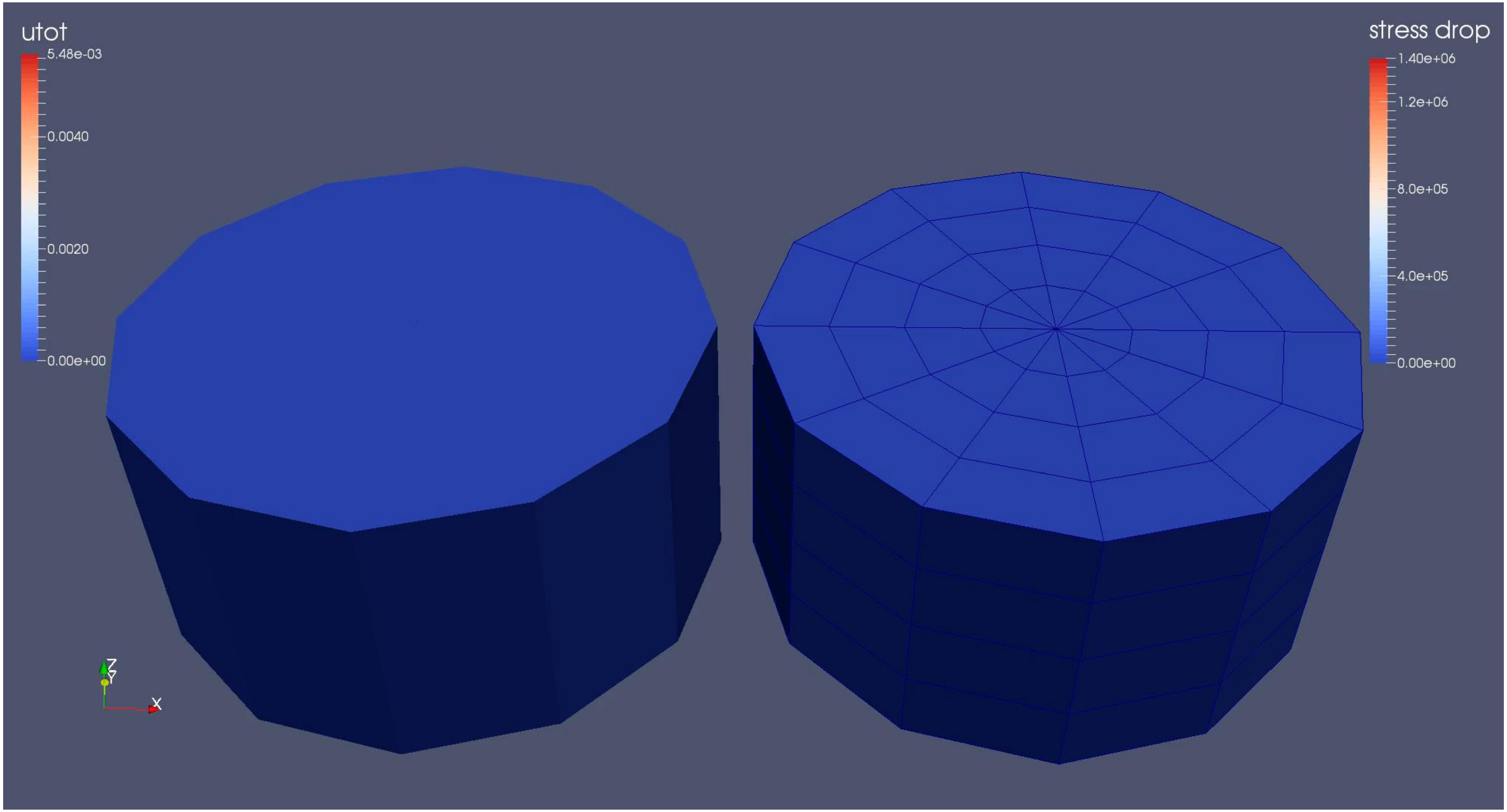
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# Conclusions & Outlook

- Combined hydro- and geomechanical model
- Phenomenological approach to failure without resolving fracture geometries in detail
  - Mohr-Coulomb criterion
  - Viscoelasticity as an equivalent to the shear stress reduction
  - Permanent effect

- Further research:
  - Expanding the approach towards tensile failure
  - Incorporate the effect of failure on the fluid transport properties

# Thank you for your attention!

## References:

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