Handbook

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http://dumux.org

Contents

1 Introduction

DuMu^{x} aims to be a generic framework for the simulation of multiphase fluid flow and transport processes in porous media using continuum mechanical approaches. At the same time, DuMu^{x} aims to deliver top-notch computational performance, high flexibility, sound software architecture and the ability to run on anything from single processor systems to highly parallel supercomputers with specialized hardware architectures.

The means to achieve these somewhat contradictory goals are the thorough use of object-oriented design in conjunction with template programming. These requirements call for C++ as the implementation language.

One of the more complex issues when dealing with parallel continuum models is managing the grids used for the spatial discretization of the physical model. To date, no generic and efficient approach exists for all possible cases, so DuMu^{x} is built on top of DUNE, the **D**istributed and **U**nified **N**umerics **E**nvironment [10]. DUNE provides a generic interface to many existing grid management libraries such as UG [14], ALUGrid [4, 3], and a few more. DUNE also extensively uses template programming in order to achieve minimal overhead when accessing the underlying grid libraries[1].

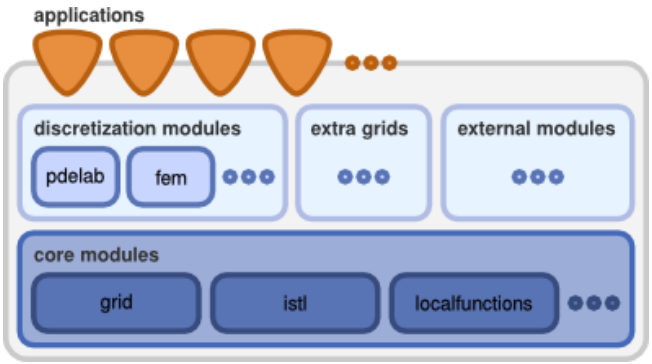


Figure 1.1: A high-level overview of DUNE's design is available on the project's web site [10].

DUNE's grid interface is independent of the spatial dimension of the underlying grid. For this purpose, it uses the concept of co-dimensional entities. Roughly speaking, an entity of co-dimension 0 constitutes a cell, co-dimension 1 entities are faces between cells, co-dimension 2 are edges, and so on until co-dimension n which are the cell's vertices. The DUNE grid interface generally assumes that all entities are convex polytopes, which means that it must be possible to express each entity as the convex hull of a set of vertices. For the sake of efficiency, all entities are further expressed in terms of so-called reference elements, which are transformed to the actual spatial incarnation within the grid by a so-called geometry function. Here, a reference element for an entity can be thought of as a prototype for the actual grid entity. For example, if we used a grid that applied hexahedrons as cells,

[1] In fact, the performance penalty resulting from the use of DUNE's grid interface is usually negligible [8].

the reference element for each cell would be the unit cube $[0, 1]^3$, and the geometry function would scale and translate the cube so that it matches the grid's cell. A quick overview of reference elements and the related numbering can be obtained from the DUNE cheat sheet (`https://www.dune-project.org/pdf/dune-cheat-sheet.pdf`). For a more thorough description of DUNE's grid definition, see [5].

In addition to the grid interface, DUNE also provides quite a few additional modules, of which the `dune-localfunctions` and `dune-istl` modules are the most relevant in the context of this handbook. `dune-localfunctions` provides a set of generic finite element shape functions, while `dune-istl` is the **I**terative **S**olver **T**emplate **L**ibrary and provides generic, highly optimized linear algebra routines for solving the generated systems.

DuMu^{x} comes in the form of an additional module `dumux`. It depends on the DUNE core modules `dune-common`,`dune-geometry`, `dune-grid`, `dune-istl`, and `dune-localfunctions`. The main intention of DuMu^{x} is to provide a framework for easy and efficient implementation of new physical models for porous media flow problems, ranging from problem formulation and the selection of spatial and temporal discretization schemes as well as nonlinear solvers, to general concepts for model coupling. Moreover, DuMu^{x} includes ready-to-use numerical models and a few example applications.

This document is the handbook to a new minor version update of DuMu^{x}: version 3.5. The release contains improvements and new features compared to version 3.4. The update is backwards compatible with the last release 3.4. To facilitate the transition for our users, we have created a changelog helping to update programs from version 3.4 to version 3.5 and giving an overview of new capabilities. It is available online at `https://git.iws.uni-stuttgart.de/dumux-repositories/dumux/blob/master/CHANGELOG.md`. We highly recommend all our users to transition with us to the most recent version of DuMu^{x} and wish everyone an exciting simulation experience.

2 Quick Start

In this chapter, we provide a quick start guide to your first DuMu^x experience, including an install script with all necessary instructions on how to very quickly install the latest release version of DuMu^x. You should have a recent working Linux environment. If you need more information, please have a look at the detailed installation instructions in chapter 3.

2.1 Prerequisites

For this quick start guide the following software packages are required:

- GitLab client
- A standard-compliant C++17 compiler supporting the common supported C++11/C++14/C++17 feature set of GCC 9.3 and Clang 10
- CMake 3.13 or newer
- pkg-config
- ParaView (to visualize the results)

2.2 Obtaining code and configuring all modules with a script

To easily install Dumux, we've provided a python-script `installdumux.py` that facilitates setting up a DUNE/DuMu^x directory tree and configures all modules with CMake. First, you will have to download this script. To do this, first navigate to the directory where you want to install DuMu^x and DUNE (a single root folder `DUMUX` will be produced, so you do not need to provide one). Then use `wget` to download the script with the following command:

```
$ wget https://git.iws.uni-stuttgart.de/dumux-repositories/dumux/-/raw/releases/3.5/bin/
    installdumux.py
```

After the download is complete, execute the script. This can be done with

```
$ python installdumux.py
```

This script will download each of the DUNE-modules that DuMu^x depends on at their 2.8 release, as well as DuMu^x's 3.5 release. After these individual modules are downloaded, DUNE and DuMu^x are automatically configured using the command-line script `dunecontrol`, located in `dune-common/bin/dunecontrol`, optimized using configure options defined in the `cmake.opts` file located in `dumux/cmake.opts`. The commands beneath the `# run dunecontrol` in the `installdumux.py` script execute this configuration script, so no further steps are required. More details about the build-system can be found in section 3.2.

2.2.1 A first test run of DuMu^x

When the `installdumux.py` script from the subsection above has run successfully, you run a simple test to ensure that the installation ran correctly. To do this, you can begin by compiling an example problem and running the simulation. You can then visualize the results using ParaView. We recommend looking at a basic single-phase groundwater flow problem.

1. Navigate to this test in the build directory using:

```
cd DUMUX/dumux/build-cmake/test/porousmediumflow/1p/isothermal
```

2. Build the executable with the following command:

```
make test_1p_tpfa
```

3. Run the simulation with the parameters listed in `params.input` like this:

```
./test_1p_tpfa params.input
```

4. Visualize the results in paraview with:

```
paraview *pvd
```

5. Advance ParaView to the next frame (green arrow button) and rescale to data range (green double arrow on top left) to admire the colorful pressure distribution.

For further information on how to get started with dumux, see section 4.

3 Detailed Installation, Documentation, and Externals

In this chapter, we provide more detailed information on how to obtain source code, build and test DUNE and DuMu^x. It further contains information on how to build the documentation and about external libraries and modules. Installing DuMu^x means that you first unpack DUNE and DuMu^x in a root directory (section 3.1). In a second step of the installation, all modules are configured with CMake (section 3.2). After the successful installation of DuMu^x, we guide you to start a test application, described in section 3.3. In section 3.4, we explain how to build the DuMu^x documentation. Lastly, section 3.5 provides details on optional libraries and modules.

In a technical sense, DuMu^x is a module of DUNE. Thus, the installation procedure of DuMu^x is the same as that of DUNE. Details regarding the installation of DUNE are provided on the DUNE website [10].

3.1 Obtaining Source Code for DUNE and DuMu^x

The DuMu^x release and trunk (developer tree) are based on the most recent DUNE release 2.8, comprising the core modules `dune-common`, `dune-geometry`, `dune-grid`, `dune-istl` and `dune-localfunctions`. For working with DuMu^x, these modules are required. All DUNE modules, including the DuMu^x module, get extracted into a common root directory, as it is done in a typical DUNE installation. We usually name our root directory `DUMUX`, but an arbitrary name can be chosen. Source code files for each DUNE module are contained in their own sub-directory within the root directory. The sub-directories for the modules are named after the module names (depending on how the modules were obtained, a version number is added to the module name). The name of each DUNE module is defined in the file `dune.module`, which is in the root directory of the respective module. This should not be changed by the user.

In section 2.1, we list some prerequisites for running DUNE and DuMu^x. Please check in said paragraph whether you can fulfill them before continuing.

Obtaining DUNE and DuMu^x from software repositories DUNE and DuMu^x use Git for their software repositories. To access them, you need to have a working installation of the version control software Git.

In the jargon of Git, *cloning a certain software version* means nothing more then fetching a local copy from the software repository and laying it out in the file system. In addition to the software, some more files for the use of the software revision control system itself are created. (If you have developer access to DuMu^x, it is also possible to do the opposite, i. e. to load up a modified revision of software into the software repository. This is usually termed as *commit* and *push*.)

The download procedure is done as follows: Create a common root directory, named e.g. `DUMUX` in the lines below. Then, enter the previously created directory and check out the desired modules. As

you see below, the check-out uses two different servers for getting the sources, one for DUNE and one for DuMu^x.

```
$ mkdir DUMUX
$ cd DUMUX
$ git clone -b releases/2.8 https://gitlab.dune-project.org/core/dune-common.git
$ git clone -b releases/2.8 https://gitlab.dune-project.org/core/dune-geometry.git
$ git clone -b releases/2.8 https://gitlab.dune-project.org/core/dune-grid.git
$ git clone -b releases/2.8 https://gitlab.dune-project.org/core/dune-istl.git
$ git clone -b releases/2.8 https://gitlab.dune-project.org/core/dune-localfunctions.git
$ git clone -b releases/3.5 https://git.iws.uni-stuttgart.de/dumux-repositories/dumux.git
```

The newest and maybe unstable developments of DUNE and DuMu^x are also provided in these repositories and can be found in the *master* branch. Please check the DUNE website [10] for further information on the DUNE development. We always try to keep up with the latest developments of DUNE. However, the current DuMu^x release is based on the stable 2.8 release and it might not compile without further adaptations using the newest versions of DUNE.

3.2 Building DUNE and DuMu^x

Configuring DUNE and DuMu^x is done by the shell-command `dunecontrol`, which is part of the DUNE build system. If you are interested in more details about the build system that is used, they can be found in the DUNE build system documentation[1] and CMake's documentation[2]. If something fails during the execution of `dunecontrol`, feel free to report it to the DUNE or DuMu^x developer mailing list, but please include error details.

It is possible to compile DuMu^x without explicitly passing options to the build system. However, this will usually not result in the most performant code. A set of default options that will usually work well is provided with DuMu^x in a file and can be passed to `dunecontrol`. The following command will configure DUNE and DuMu^x and build all necessary libraries

```
$ # make sure you are in the common root directory
$ ./dune-common/bin/dunecontrol --opts=dumux/cmake.opts all
```

If you are going to compile with modified options, the following can be a starting point:

```
$ # make sure you are in the common root directory
$ cp dumux/cmake.opts my-cmake.opts  # create a personal version
$ gedit my-cmake.opts                # optional editing the options file
$ ./dune-common/bin/dunecontrol --opts=my-cmake.opts all
```

Sometimes, it is necessary to have additional options which are specific to a package set of an operating system or sometimes you have your own preferences. Feel free to work with your own set of options, which may change over time. The option file, that comes with the distribution, is to be understood more as a starting point for setting up own customization than as something which is

[1] `https://www.dune-project.org/buildsystem/`
[2] `https://cmake.org/documentation/`

fixed. The use of external libraries can make it necessary to add quite many options in an option file. It can be helpful to give your customized option file a unique name, as done above, to avoid confusing it with the option files which came out of the distribution.

3.3 The First Run of a Test Application

We recommend the "Getting Started" guide on the DuMu^x website: `https://dumux.org/gettingstarted/`. Nevertheless, here is a brief description of how to run a first application.

The previous section showed how to install and compile DuMu^x. This section shall give a very brief introduction on how to run a first test application and how to visualize the first output files.

All executable files are compiled in the `build` sub-directories of DuMu^x. If not specified differently in the options file, this is `build-cmake` as default.

1. Enter the folder `test/porousmediumflow/2p/incompressible` within your build directory. Type `make test_2p_incompressible_tpfa` in order to compile the application `test_2p_incompressible_tpfa`. To run the simulation, type `./test_2p_incompressible_tpfa params.input` into the console. The added `params.input` specifies that all important run-time parameters (such as first time step size, end of simulation and location of the grid file) can be found in a text file in the same directory with the name `params.input`.

2. The simulation starts and produces some VTU output files and also a PVD file. The PVD file can be used to examine time series and summarizes the VTU files. (It is possible to abort a running application by pressing <Ctrl><c>.)

3. You can display the results using the visualization tool ParaView (or alternatively VisIt). Just type `paraview` in the console and open the PVD file. On the upper left-hand side, you can choose the desired parameter to be displayed.

3.4 Building Documentation

Building the included documentation like this handbook requires LaTeX and the auxiliary tool `bibtex`. One usually chooses a LaTeX distribution like `texlive` for this purpose. It is possible to switch off building the documentation by setting the switch `--disable-documentation` in the `CONFIGURE_FLAGS` of the building options, see section 3.2.

3.4.1 Doxygen

Doxygen documentation is done by specifically formatted comments included in the source code, which can get extracted by the program `doxygen`. Besides extracting these comments, `doxygen` builds up a web-browsable code-structure documentation like class hierarchy of code displayed as graphs, see `https://www.doxygen.nl/index.html`.

The Doxygen documentation of a module can be built by running `dunecontrol` (if `doxygen` is installed), entering the `build-*`directory, and executing `make doc`. Then point your web browser to the file `MODULE_BUILD_DIRECTORY/doc/doxygen/html/index.html` to read the generated documentation. This should also work for other DUNE modules.

3.4.2 Handbook

To build the DuMu^x handbook, navigate to the `build`-directory and run `make doc` or `make doc_handbook_0_dumux-handbook_pdf`. The pdf can then be found in `MODULE_BUILD_DIRECTORY/doc/handbook/0_dumux-handbook.pdf`.

3.5 External Libraries and Modules

The libraries described below provide additional functionality but are not generally required to run DuMu^x. If you are going to use an external library, check the information provided on the DUNE website[3]. You can consult the DUNE website for higher-level DUNE modules, such as Grid Modules [4], Discretization Modules [5], and Extension Modules [6].

Installing an external library can require additional libraries which are also used by DUNE. For some libraries, such as BLAS or MPI, multiple versions can be installed on the system. Make sure that it uses the same library as DUNE when configuring the external library.

Some of the libraries are then compiled within that directory and are not installed in a different place, but DUNE may need to know their location. Thus, one may have to refer to them as options for `dunecontrol`, for example via the options file `my-cmake.opts`. Make sure you compile the required external libraries before you run `dunecontrol`. An easy way to install some of the libraries and modules, given below, is the `installexternal.py` script located in `bin`. The script has to be called from your common root directory.

3.5.1 List of External Libraries and Modules

In the following list, you can find some external modules and external libraries, and some more libraries and tools which are prerequisites for their use.

- **dune-ALUGrid**: Grid library, comes as a DUNE module. The parallel version needs also a graph partitioner, such as ParMETIS. Download: `https://gitlab.dune-project.org/extensions/dune-alugrid`
- **dune-foamgrid**: External grid module. One- and two-dimensional grids in a physical space of arbitrary dimension; non-manifold grids, growth, element paramterizations, and movable vertices. This makes FoamGrid the grid data structure of choice for simulating structures such as foams, discrete fracture networks, or network flow problems. Download: `https://gitlab.dune-project.org/extensions/dune-foamgrid`
- **opm-grid**: opm-grid is a DUNE module supporting grids in a corner-point format. Download: `https://github.com/OPM/opm-grid.git` and `https://github.com/OPM/opm-common.git`
- **dune-subgrid**: The dune-subgrid module is a meta-grid implementation that allows to mark elements of another hierarchical dune grid and use this sub-grid just like a regular grid. The

[3] DUNE: External libraries, `https://www.dune-project.org/doc/external-libraries/`
[4] DUNE: Grid modules, `https://www.dune-project.org/groups/grid/`
[5] DUNE: Discretization modules, `https://www.dune-project.org/groups/disc/`
[6] DUNE: Extension modules, `https://www.dune-project.org/groups/extension/`

set of marked elements can then be accessed as a hierarchical dune grid in its own right. Dune-Subgrid provides the full grid interface including adaptive mesh refinement. Download: `https://git.imp.fu-berlin.de/agnumpde/dune-subgrid.git`

- **dune-spgrid**: The DUNE module dune-spgrid provides a structured, parallel grid and supports periodic boundary conditions. Download: `https://gitlab.dune-project.org/extensions/dune-spgrid.git`
- **SuperLU**: External library for solving linear equations. SuperLU is a general purpose library for the direct solution of large, sparse, non-symmetric systems of linear equations. Download: `http://crd.lbl.gov/~xiaoye/SuperLU`
- **UMFPack**: External library for solving linear equations. It is part of SuiteSparse. See: `http://faculty.cse.tamu.edu/davis/suitesparse.html`. On Debian/Ubuntu you can install the package `libsuitesparse-dev`.
- **dune-UG**: External library for use as grid. UG is a toolbox for unstructured grids, released under GPL. To build UG the tools `lex/yacc` or the GNU variants of `flex/bison` must be provided. Download: `https://gitlab.dune-project.org/staging/dune-uggrid`

The following are dependencies of some of the used libraries. You will need them depending on which modules of DUNE and which external libraries you use.

- **MPI**: The parallel version of DUNE and also some of the external dependencies need MPI when they are going to be built for parallel computing. `OpenMPI` and `MPICH` in a recent version have been reported to work.
- **BLAS**: SuperLU makes use of BLAS. Thus install GotoBLAS2, ATLAS, non-optimized BLAS or BLAS provided by a chip manufacturer. Take care that the installation scripts select the intended version of BLAS.
- **METIS** and **ParMETIS**: This are dependencies of ALUGrid and can be used with UG, if run in parallel.
- **Compilers**: Beside `g++`, DUNE can be built with Clang from the LLVM project and Intel C++ compiler. C and Fortran compilers are needed for some external libraries. As code of different compilers is linked together, they have to be be compatible with each other.

3.6 Backwards Compatibility

Dumux Releases are split into major (e.g. 2.0, 3.0) and minor (e.g. 3.1, 3.2, 3.3, 3.4, 3.5) releases. Major releases are not required to maintain backwards compatibility but would provide a detailed guide on updating dependent modules. For each minor release, maintaining backwards compatibility is strongly encouraged and recommended.

Maintaining backwards compatibility means for all changes made to the dumux master, each test and all dumux dependent modules should still compile. In addition, the user should be warned at compile time of any relevant interface changes. This can be done by deprecating the old method

with a deprecation message and forwarding it to the new method. Examples of this are shown in the contribution guide. Each of these deprecation messages should also include the release in which the interface will be removed, and all changes should be documented thoroughly in the changelog.md.

Despite the goal of maintaining backwards compatibility across minor releases, for more complicated changes, this is to be decided upon on a case-by-case basis, due to limited developer resources. In the case that implementing full backwards compatibility for an update is not feasible, or would require unreasonable resources, the degree of backwards compatibility will be decided by a vote in one of the monthly core developer meetings.

4 Learning to use DuMu^x

So, you've downloaded your very own copy of DuMu^x and its dependencies. You've run dunecontrol, and your example "test_dumux" not only compiles, but it even shows a nice simulation in ParaView. Maybe you've read through parts of the handbook, and even started looking through the Doxygen documentation. Well done. What now?

"How on earth is this going to help me solve my multi-(phase, component, scale, physics) flow and transport problems in porous media systems?", you begin to wonder. Don't panic! In order to best ease our prospective users and developers into the wonderful DuMu^x simulation environment, we've prepared a DuMu^x course and extensively-documented examples.

4.1 Hands-on DuMu^x experience – the DuMu^x course

This course is offered once a year over a period of 3 days at the University of Stuttgart. If you're looking for information on attending, subscribe to the DuMu^x mailing list and stay tuned for updates: `https://listserv.uni-stuttgart.de/mailman/listinfo/dumux`.

"But the course won't take place for another 6 months!" and, *"I want to start developing a numerical model of my challenging and interesting process now!"*, you think. Not a problem. The course materials are all shared online in their own git repository. A series of beginner-level exercises are explained such that you can see how a model is developed in DuMu^x. As a teaser, we've also included a suite of examples from hot topics we're working on. Models exploring "Coupling free flow and porous-media flow", "Flow in fractured porous media" and "Fluid-solid phase change" are all introduced.

"Sounds great, but where is this material? I can't find it within what I've downloaded.", you question. The DuMu^x course material is available online: `https://git.iws.uni-stuttgart.de/dumux-repositories/dumux-course`.

In order to download this repository, which acts as an additional module to the DuMu^x base, you can download an installation script with the following command:

```
$ wget https://git.iws.uni-stuttgart.de/dumux-repositories/dumux-course/raw/releases/3.5/
    scripts/install.sh
```

This script will install `dumux`, it's Dune dependencies, and the `dumux-course` repository. Within the directory `dumux-course` there are a series of exercises and slides describing the previously described examples. The course can also be added to your existing dumux installation using the `installExternal.py` script and the argument `course`.

The DuMu^x course will be updated with each DuMu^x release. The above script will download the correct version (**releases/3.5**) of both the `dumux` and `dumux-course` module.

4.2 Experience DuMu^x by reading – the DuMu^x examples

As an alternative to going through exercises, you can have a look at our well-documented DuMu^x examples in DuMu^x's `examples` directory. They show how to apply DuMu^x models to typical physical problems. In the `README.md` files, the setup is explained, the used code is presented and documented and images resulting from the simulation are included. The `README.md` files are located directly in the subfolders of `examples` and can be displayed by web browsers. The number of examples is continuously growing. You can find a list of current examples under: `https://git.iws.uni-stuttgart.de/dumux-repositories/dumux/-/tree/releases/3.5/examples`.

4.3 Further Practice

If there is a need for further practice, we refer here to the test problems that are already implemented in DuMu^x. Several examples for all models can be found in the `test`-directory.

Another possibility to gain more experience with DuMu^x is the `dumux-lecture` module that contains different application examples that are used in the lectures at the Department of Hydromechanics and Modelling of Hydrosystems in Stuttgart. The `dumux-lecture` module can be obtained with the `installExternal.py` script and the argument `lecture`.

The module is structured based on the different lectures:

- mm: Multiphase Modelling,
- efm: Environmental Fluid Mechanics,
- mhs: Modelling of Hydrosystems.

The majority of applications are covered in the course Multiphase Modelling (mm), while there are also some basic examples in the courses Environmental Fluid Mechanics (efm) and Modelling of Hydrosystems (mhs). These applications are primarily designed to enhance the understanding of conceptualizing the governing physical processes and their implementation in a numerical simulator. Different aspects of modeling multi-phase multi-component flow and transport processes are shown. The lectures focus on questions such as the assignment of boundary conditions, the choice of the appropriate physics for a given problem (which phases, which components), discretization issues, time stepping. You can find, e. g., a comparison of different two-phase flow problems: The simpler approach considers two immiscible fluids while components in both phases with inter-phase mass transfer are considered in the more complex approach. All scenarios and their physical background are explained in additional .tex-files, which are provided in sub-directories named `description`. The following test cases are contained in the `dumux-lecture` module:

- `buckleyleverett`: The Buckley-Leverett Problem is a classical porous media flow show case
- `co2plume`: Analysis of the influence of the gravitational number on a CO_2 plume
- `columnxylene`: An experiment of the Research Facility for Subsurface Remediation, University of Stuttgart
- `convectivemixing`: A test case related to CO_2 storage

- `fractures`: Two-phase flow in fractured porous media
- `fuelcell`: Water management in PEM fuel cells
- `heatpipe`: A show case for two-phase two-component flow with heat fluxes
- `heavyoil`: Steam assisted gravity drainage (SAGD)
- `henryproblem`: A show case related to salt water intrusion
- `mcwhorter`: The McWhorter Problem is a classical porous media flow show case
- `naplinfiltration`: Infiltration of non-aqueous phase liquid (NAPL) into soil
- `remediationscenarios`: Test case for NAPL contaminated unsaturated soils
- `groundwater`: Simple groundwater flow case for the course Modelling of Hydrosystems (mhs)
- Different single/two-phase, single/two-component problems: Examples from the course Environmental Fluid Mechanics (efm)

5 Overview and Infrastructure

This chapter provides an overview of the general structure in DuMu^x (5.1) and gives help for basic work with DuMu^x (5.2-5.5). Further, it presents useful external tools (5.6) and basic concepts (5.8).

5.1 Directory Structure

DuMu^x has the following folder structure, which is similar to other DUNE modules.

- `bin`: binaries, e.g. used for the automatic testing, post-processing, installation
- `cmake`: the configuration options for building DuMu^x
- `doc`: files necessary for the Doxygen documentation and this handbook, and various logos
- `dumux`: the main folder, containing the source files. See Fig. 5.1 for a visualized structure. For more information on the models, have a look at the Doxygen documentation.
- `examples`: well-documented examples of applying DuMu^x to typical physical problems. In the `README.md` files, the setup is explained, the used code is presented as well as documented and images resulting from the simulation are included. The `README.md` files are located directly in the subfolders of `examples` and can be displayed by web browsers.
- `test`: tests for each numerical model and some functionality. The structure is equivalent to the `dumux` folder, the `references` folder contains solutions for the automatic testing. Each test program consist of a main file `main.cc`, the problem definition `*problem.hh` (specifying initial and boundary conditions), and an input file `params.input`. If necessary, spatially dependent parameters are defined in `*spatialparameters.hh`. For more detailed descriptions of the tests, please have a look at the Doxygen documentation.

5.2 Setup of new Folders and new Tests

This section describes how to set up a new folder and how to tell the build system there is a new one.

Adding new Folders

1) create new folder with content
2) adapt the `CMakeList.txt` in the folder above by adding a line with `add_subdirectory(NEW_FOLDER)`
3) create a `CMakeList.txt` in the newly created folder
4) go to your `build`-directory and type `make` to re-configure the system

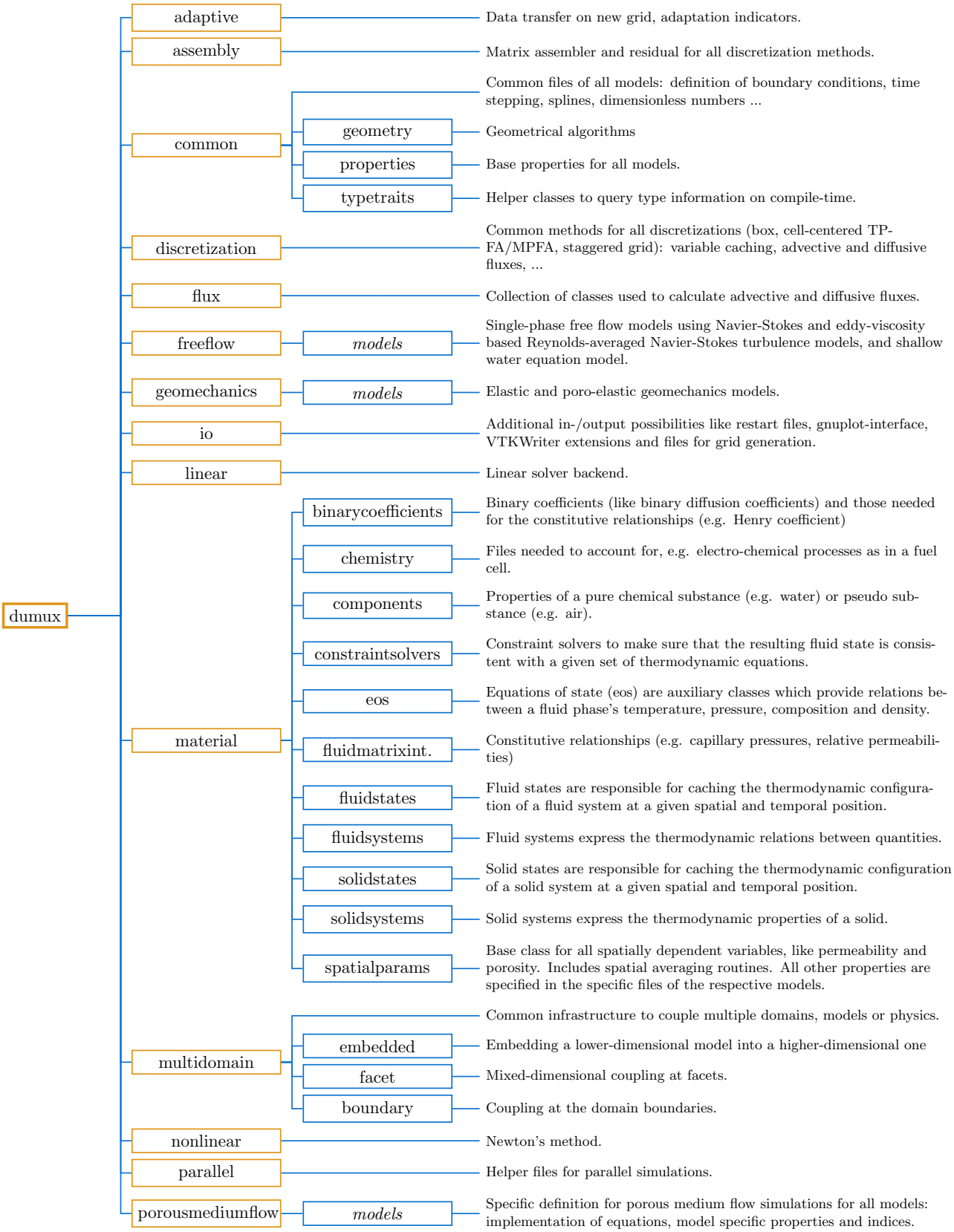


Figure 5.1: Structure of the directory `dumux` containing the DuMu^{x} source files.

Adding new Test Programs To add a test use the `add_dune_test` macro within the `CMakeList.txt` file. The macro can be used with a variable amount of arguments. A simple call could look like this:

```
dumux_add_test(NAME my_test
               SOURCES main.cc
               CMD_ARGS my_test params.input)
```

Here, we create an executable called `my_test` from a source file `main.cc`. The name of the test will also be `my_test` (has to be unique). The last argument specifies a command - here, we just run the executable `my_test` with an input file `params.input`. For more advanced uses of the `add_dune_test` macro, have a look at the `test` directory. A complete documentation is given under `https://www.dune-project.org/sphinx/core-2.7/`.

5.3 Parameters in DuMu^x

Simulation parameters can be parsed to the program via a parameter file or via the command line.

After having run the example application from section 3.3 you will get the following output at the end of the simulation run [1]:

```
# Runtime-specified parameters used:
[ Grid ]
Cells = "48 32"
UpperRight = "6 4"
[ Newton ]
EnablePartialReassembly = "true"
[ Problem ]
EnableGravity = "true"
Name = "2p"
[ SpatialParams ]
LensLowerLeft = "1.0 2.0"
LensUpperRight = "4.0 3.0"
[ TimeLoop ]
DtInitial = "250"
TEnd = "3000"

# Global default parameters used:
[ Assembly ]
NumericDifferenceMethod = "1"
[ Flux ]
UpwindWeight = "1.0"
[ LinearSolver ]
MaxIterations = "250"
ResidualReduction = "1e-13"
Verbosity = "0"
[ LinarSolver.Preconditioner ]
Iterations = "1"
Relaxation = "1.0"
[ Newton ]
EnableAbsoluteResidualCriterion = "false"
```

[1] If you did not get the output, add `Parameters::print();` to your main file.

```
EnableChop = "false"
EnableResidualCriterion = "false"
EnableShiftCriterion = "true"
MaxAbsoluteResidual = "1e-5"
MaxRelativeShift = "1e-8"
MaxSteps = "18"
MinSteps = "2"
ResidualReduction = "1e-5"
SatisfyResidualAndShiftCriterion = "false"
TargetSteps = "10"
UseLineSearch = "false"
[ TimeLoop ]
MaxTimeStepSize = "1e300"
[ Vtk ]
AddProcessRank = "true"
AddVelocity = "false"

# Unused parameters:
Grid.LowerLeft = "0 0"
```

A number of things can be learned:

- *run-time* parameters can be changed without re-compiling
- *default parameters* are set by default
- *unused* parameters are not used by the simulation (maybe typo or wrong group in input file)

5.3.1 Parameter Values

To get the value of an input parameter please use:

```
static const TYPE paramname = getParam<TYPE>("GROUPNAME.PARAMNAME");
```

If you also want to set a default value for a parameter, just add it like this:

```
static const TYPE paramname = getParam<TYPE>("GROUPNAME.PARAMNAME", default);
```

As this function call is relatively expensive, the respective variables should always be `static` (e.g., if used in a loop). When dealing with multiple group names, e.g., in the context of coupled models, the following methods might be more convenient:

```
auto modelParamGroup0 = "Model0";
static const TYPE paramname0 = getParamFromGroup<TYPE>(modelParamGroup0, "GROUPNAME.PARAMNAME");
auto modelParamGroup1 = "Model1";
static const TYPE paramname1 = getParamFromGroup<TYPE>(modelParamGroup1, "GROUPNAME.PARAMNAME");
```

The `FVProblem` class provides a convenience function `paramGroup()`.
The parameters can then be specified in the input file:

```
[ Model0.Grid ]
File = file0.dgf
[ Model1.Grid ]
File = file1.dgf
```

5.4 Restart DuMu^x Simulations

DuMu^x has some experimental support for check-pointing (restarting paused/stopped/crashed simulations). You can restart a DuMu^x simulation from any time point where a VTK file was written out. This is currently only supported for sequential, non-adaptive simulations. For adaptive simulation the full hierarchical grid has to be stored. This is usually done with the grid's `BackupRestoreFacility`. There is currently no special support by DuMu^x for that, but it is possible to implement a restart using `BackupRestoreFacility` with plain Dune.

For VTK files the output can be read with the free function `loadSolution`. Grids can be read with the `Dumux::VTKReader` or you can simply recreate the grid as you did in the first simulation run.

Writing double-precision floating point numbers to VTK files is available since DUNE release 2.7. If you are using that version, it is now possible to specify output precision in the input file using `Vtk.Precision` followed by either `Float32`, `Float64`, `UInt32`, `UInt8` or `Int32`. `Float32` is set as the default. We especially advice the use of `Float64` when working with restart files.

The restart capabilities will hopefully be improved in future versions of DuMu^x-3. We are looking forward to any contributions (especially HDF5 / XDMF support, improvement of VTK support).

5.5 Developing DuMu^x

5.5.1 Communicate with DuMu^x Developers

Issues and Bug Tracking The bug-tracking system *GitLab Issues* offers the possibility to report bugs or discuss new development requests. Feel free to register (if you don't have an account at out *Git* yet) and to contribute at `https://git.iws.uni-stuttgart.de/dumux-repositories/dumux/issues`.

Commits, Merges, etc. To be up-to-date with the latest changes made to any git-repository, you can use RSS Feeds. Simply click on *Issues* or *Activity* and then select a tab you are interested in and use your favorite RSS-application for receiving the news.

Automatic Testing Dashboard The automatic testing using *BuildBot* helps to constantly check the DuMu^x problems for compiling and running correctly. It is available at `https://git.iws.uni-stuttgart.de/buildbot/#/builders`.

The General Mailing List: If you have questions, specific problems (which you really struggle to solve on your own), or hints for the DuMu^x-developers, please contact the mailing list `dumux@iws.uni-stuttgart.de`. You can subscribe to the mailing list via `https://listserv.uni-stuttgart.de/mailman/listinfo/dumux`, then you will be informed about upcoming releases or events.

5.5.2 Coding Guidelines

Writing code in a readable manner is very important, especially for future code developers (e.g. for adding features, debugging, etc.). For the style guide and instructions how to contribute to DuMu^x visit `https://git.iws.uni-stuttgart.de/dumux-repositories/dumux/blob/master/CONTRIBUTING.md`.

5.5.3 Tips and Tricks

DuMu^x users and developers at the LH2 are also referred to the internal confluence pages for more information.

Optimized computation vs debugging DUNE and DuMu^x are built with the help of `dunecontrol`, as explained on page 8. Per default, DuMu^x is compiled using optimization options, which leads to faster runtimes but is unsuitable for debugging. For debug opts you can set `DCMAKE_BUILD_TYPE` to `Debug` or `RelWithDebInfo` in your options file. You can also do this in any of the `CMakeLists.txt` in Dumux by adding:

```
set(CMAKE_BUILD_TYPE Debug)
```

Afterwards rerun cmake (run cmake <`path-to-build-dir`>).

Dunecontrol for selected modules A complete build using `dunecontrol` takes some time. In many cases not all modules need to be re-built. Pass the flag `--only=dumux` to `dunecontrol` for configuring or building only DuMu^x. A more complex example would be a case in which you have to configure and build only e.g. DUNE-grid and DuMu^x. This is achieved by adding `--only=MODULE,dumux`.

Patching Files or Modules If you want to send changes to an other developer of DuMu^x providing patches can be quite smart. To create a patch simply type:

```
$ git diff > PATCHFILE
```

which creates a text file containing all your changes to the files in the current folder or its subdirectories. To apply a patch in the same directory type:

```
$ patch -p1 < PATCHFILE
```

File Name and Line Number by Predefined Macro If you want to create output in order to later know where some output or debug information came from, use the predefined macros `__FILE__` and `__LINE__`:

```
std::cout << "# This was written from "<< __FILE__ << ", line " << __LINE__ << std::endl;
```

Using DUNE Debug Streams DUNE provides a helpful feature for keeping your debug-output organized. It uses simple streams like `std::cout`, but they can be switched on and off for the whole project. You can choose five different levels of severity:

```
5 - grave (dgrave)
4 - warning (dwarn)
3 - info (dinfo)
2 - verbose (dverb)
1 - very verbose (dvverb)
```

They are used as follows:

```
// define the minimal debug level somewhere in your code
#define DUNE_MINIMAL_DEBUG_LEVEL 4
Dune::dgrave << "message"; // will be printed
Dune::dwarn << "message"; // will be printed
Dune::dinfo << "message"; // will NOT be printed
```

Make headercheck: To check one header file for all necessary includes to compile the contained code, use `make headercheck`. Include the option `-DENABLE_HEADERCHECK=1` in your opts file and run `dunecontrol`. Then go to the top level in your build-directory and type `make headercheck` to check all headers or press 'tab' to use the auto-completion to search for a specific header.

5.6 External Tools

5.6.1 Git

Git is a version control tool which we use. The basic Git commands are:

- `git checkout`: receive a specified branch from the repository
- `git clone`: clone a repository; creates a local copy
- `git diff`: to see the actual changes compared to your last commit
- `git pull`: pull changes from the repository; synchronizes the repository with your local copy
- `git push`: push comitted changes to the repository; synchronizes your local copy with the repository
- `git status`: to check which files/folders have been changed
- `git gui`: graphical user interface, helps selecting changes for a commit

5.6.2 Gnuplot

A gnuplot interface is available to plot or visualize results during a simulation run. This is achieved with the help of the `Dumux::GnuplotInterface` class provided in `io/gnuplotinterface.hh`.

To use the gnuplot interface you have to make some modifications in your file, e.g., your main file. First, you have to include the corresponding header file for the gnuplot interface.

```
#include <dumux/io/gnuplotinterface.hh
```

Second, you have to define an instance of the class `Dumux::GnuplotInterface` (e.g. called `gnuplot`).

```
Dumux::GnuplotInterface<double> gnuplot;
```

As an example, to plot the mole fraction of nitrogen (`y`) over time (`x`), extract the variables after each time step in the time loop. The actual plotting is done using the method of the gnuplot interface:

```
gnuplot.resetPlot();                        // reset the plot
gnuplot.setXRange(0.0, 72000.0);            // specify xmin and xmax
gnuplot.setYRange(0.0, 1.0);                // specify ymin and ymax
gnuplot.setXlabel("time [s]");              // set xlabel
gnuplot.setYlabel("mole fraction mol/mol"); // set ylabel

```

```
// set x-values, y-values, the name of the data file and the Gnuplot options
gnuplot.addDataSetToPlot(x, y, "N2.dat", options);

gnuplot.plot("mole_fraction_N2");           // set the name of the output file
```

It is also possible to add several data sets to one plot by calling `addDataSetToPlot()` more than once. For more information have a look into a test including the gnuplot interface header file, the doxygen documentation of `Dumux::GnuplotInterface`, or the header file itself (`dumux/io/gnuplotinterface.hh`).

5.6.3 Gstat

Gstat is an open source software tool which generates geostatistical random fields (see `www.gstat.org`). In order to use gstat, execute the `bin/installexternal.py` from your DuMu^{x} root directory or donwload, unpack and install the tarball from the gstat-website. Then, rerun cmake (in the second case set `GSTAT_ROOT` in your input file to the path where gstat is installed).

5.6.4 ParaView

To visualize the simulation data you have produced using DuMu^{x}, we recommend using Paraview. This open-source software supports DuMu^{x} 's standard data formats, and can be operated either with a GUI or with batching tools.

5.7 Scripts

A suite of scripts is available within the dumux repository to assist in the performance of common tasks. These scripts can be found in the `dumux/bin/` directory. The majority of these scripts began as bash (*.sh) scripts, but most have been replaced with more portable and user-friendly Python versions. Some of these scripts are used in the installation or the creation of working dumux-dependent repositorities; these are outlined in section 5.7.1. Others can be used to perform post-processing tasks after simulation data has been produced; these are outlined in section 5.7.2. In order to maintain the suite of unit tests and examples within this repository, some testing scripts are also required; these are outlined in section 5.7.3. All remaining tests are mentioned in section 5.7.4.

5.7.1 Repository development Scripts:

- Installation Scripts: `installdumux.py` and `installexternal.py` both have been developed to make the installation of the base Dumux suite and any external dependencies as simple as possible.
- Module Development: When developing within the dumux simulation environment, it is recommended to do so within your own module. When sharing this module with others, or installing it on multiple machines, it is often difficult to find exactly which version you are using so that your work will run in different places. To simplify this, the following scripts are available in the `util/` directory:
  - The script `createdockerimage.py` can help to create a docker image to put your work in a container for simple delivery.

- In order to extract the content and dependencies within your model, `extractmodulepart.py` will collect all headers used to run a specified test and collect them all in a functioning external module.
- To document the exact versions that have been used, the scripts `getusedversions.py` is available to search each through an existing simulation dumux suite and find the branch, commit hash, and commit date of the current implemention.
- The script `create_cmakelists.py` can be used to integrate new header files to the dune/-dumux cmake system.
- In order to produce an installscript that will install your repository and its dependencies on another computer, the `makeinstallscript.py` is available. This script will collect all of the dependencies and their versions using the `getmoduleinfo.py` and the other scripts listed above and write a script to install a new version in the same configuration. In the past it has been recommended to run the script `test_dumux.sh` to ensure that the final dunecontrol and compilation pass have worked.

5.7.2 Post-processing Scripts:

Data visualization and post processing is an integral part of any exercise in simulation. A few scripts are available to begin with in the `postprocessing/` subdirectory.

- Paraview data collection scripts: Although there are many tools available within the open-source visualization program paraview (see 5.6.4), performing consistent data collection from your simulations can be tedious when analyzing many output files. The scripts `exportscreenshot2d.py`, `extractlinedata.py`, and `extractpointdataovertime.py` are each scripts that can help to perform paraview data collection commands consistently and each produce easy to use `.csv` files or images.

- Error convergence script: When evaluting errors to a reference solution across grid refinements, the L2 error is typically evaluated. A script to help perform this is available in `l2error.py`.

5.7.3 Testing Scripts:

In order to ensure consistency when developing in dumux, developers are encouraged to introduce unit tests and reference tests that can be run to ensure new code introductions do not break existing functions. While existing simulations should each test something when the `ctest` command is called, a few scripts exist in the `testing/` subdirectory to make this simple to develop and quick to evaluate.

- Run tests: In order to automatically find and run tests that fit a specific critera, the scripts `findtests.py`, `runtest.py` and `runselectedtests.py` each are available. These are each used to automate the testing process.

- Reference Solution Comparisons: The scripts `fuzzycomparevtu.py` and `fuzzycomparedata.py` are both used to compare simulation results against reference simulation results. When running a test, one can use these files to compare the output from the test against reference simulation results. Should these results differ, the difference for each field is shown. These scripts are typically directly called during testing.

5.7.4 Miscellaneous:

In addition, a few miscellaneous scripts are available.

- Parameter scripts: The scripts `doc/compareparameters.sh` and `doc/getparameterlist.py` are both available for tracking all of the runtime parameters used.
- Other: `doc/getcontributors.sh` can be used to find the names and contact information for all commit authors over a given time period.

5.8 Assembling the linear system

The physical system is implemented as the mathematical differential equation in local operators. DuMu^{x} generates the linear system automatically. Read on, to learn what is done internally.

5.8.1 Newton's method

The differential equations are implemented in the residual form. All terms are on the left hand side and are summed up. The terms contain values for the primary variables which are part of the solution vector $\mathbf{u}$. The sum of the terms is called residual $\mathbf{r}(\mathbf{u})$ which is a function of the solution. For example:

$$\underbrace{\phi\frac{\partial \varrho_\alpha S_\alpha}{\partial t} - \operatorname{div}\left(\varrho_\alpha \frac{k_{r\alpha}}{\mu_\alpha}\mathbf{K}\left(\mathbf{grad}\ p_\alpha - \varrho_\alpha \mathbf{g}\right)\right) - q_\alpha}_{=:\,\mathbf{r}(\mathbf{u})} = 0$$

We don't know the solution $\mathbf{u}$, so we use the iterative Newton's method to obtain a good estimate of $\mathbf{u}$. We start with an initial guess $\mathbf{u}^0$ and calculate it's residual $\mathbf{r}(\mathbf{u}^0)$. To minimize the error, we calculate the derivative of the residual with respect to the solution. This is the Jacobian matrix

$$\frac{\mathrm{d}}{\mathrm{d}\mathbf{u}}\mathbf{r}\left(\mathbf{u}^i\right) = J_{\mathbf{r}(\mathbf{u}^i)} = \left(\frac{\mathrm{d}}{\mathrm{d}\mathbf{u}^i_m}\mathbf{r}\left(\mathbf{u}^i\right)_n\right)_{m,n}$$

with i denoting the Newton iteration step. Each column is the residual derived with respect to the mth entry of $\mathbf{u}^i$.

The Jacobian indicates the direction where the residual increases. By solving the linear system

$$J_{\mathbf{r}(\mathbf{u}^i)} \cdot \mathbf{x}^i = \mathbf{r}(\mathbf{u}^i)$$

we calculate the direction of maximum growth $\mathbf{x}^i$. We subtract it from our current solution to get a new, better solution $\mathbf{u}^{i+1} = \mathbf{u}^i - \mathbf{x}^i$.

We repeat the calculation of the Jacobian $J_{\mathbf{r}(\mathbf{u}^i)}$ and the direction of maximum growth $\mathbf{x}^i$ until our approximated solution becomes good enough.

5.8.2 Structure of matrix and vectors

To understand the meaning of an entry in the matrix or the vector of the linear system, we have to define their structure. Both have a block structure. Each block contains the degrees of freedom (also

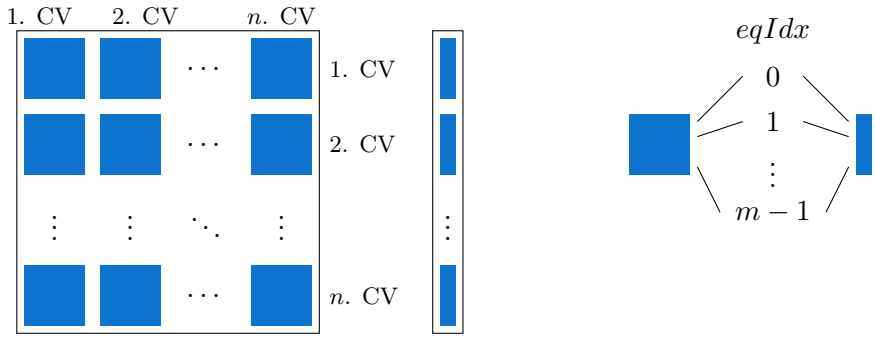


Figure 5.2: Structure of matrix and vector, left: block structure, right: within block

called variables or unknowns) for a control volume. The equation index is used to order the degrees of freedom. For each control volume we have one block. The mapper is used to order the blocks.

Accessing entries follows this structure. You can access the pressure value in the third ($n = 3$) sub-control volume in a solution vector sol with sol[n−1][pressureIdx]=sol[2][pressureIdx].

6 Advanced DuMu^{x} – Detailed Instructions

This chapter contains detailed information for those who are interested in deeper modifications of underlying DuMu^{x} models, classes, functions, etc.

6.1 Physical Basics

Here, the basic definitions, the general models concept, and a list of models available in DuMu^{x} are given. The actual differential equations can be found in the local residuals (see Doxygen documentation of the model's `LocalResidual` class).

6.1.1 Basic Definitions and Assumptions

Basic definitions and assumptions are given. More information can be found e.g. in [2, 6].

Phases: A *phase* is defined as a continuum having distinct properties (e.g. density and viscosity). If phases are miscible, they contain dissolved portions of the substance of the other phase. Fluid and solid phases are distinguished. The fluid phases have different affinities to the solid phases. The phase, which has a higher affinity to the solid phases is referred to as the (more) wetting phase. In the case of two phases, the less wetting one is called the nonwetting phase.

For compositional multi-phase models, fluid phases may be composed of several components, while the solid phases are assumed to consist exclusively of a single component.

Components: The term *component* stands for constituents of the phases which can be associated with a unique chemical species or, more generally, with a group of species exploiting similar physical behavior. For example, Fig. 6.1 shows a water-gas-NAPL system composed of the phases water (subscript w), gas (g), and NAPL (n). These phases are composed of the components water (superscript w), the pseudo-component air (a), and an organic contaminant (c).

The composition of the components in a phase can influence the phase properties. Furthermore, for mass transfer, the phase behavior is quite different from the component behavior.

Equilibrium: For the non-isothermal, multi-phase, multi-component processes in porous media we state the assumption of *local thermodynamic equilibrium*. Chemical equilibrium means that the mass/mole fractions of a component in different phases are in equilibrium. Thermal equilibrium assumes the same temperature for all considered phases. Mechanical equilibrium is not valid in a porous medium, since discontinuities in pressure can occur across a fluid-fluid interface due to capillary effects.

Notation: The subscript index α, e.g. w, n and g in the example of Fig. 6.1, refers to the phase, while the superscript κ, e.g. w, a and c in the example of Fig. 6.1, refers to the component.

p_α	phase pressure	ϕ	porosity
T	temperature	K	absolute permeability tensor
S_α	phase saturation	τ	tortuosity
x_α^κ	mole fraction of component κ in phase α	$\boldsymbol{g}$	gravitational acceleration
X_α^κ	mass fraction of component κ in phase α	q_α^κ	volume source term of κ in α
$\varrho_{\mathrm{mol},\alpha}$	molar density of phase α	u_α	specific internal energy
ϱ_α	mass density of phase α	h_α	specific enthalpy
M	molar mass of a phase or component	c_s	specific heat enthalpy
$k_{\mathrm{r}\alpha}$	relative permeability	λ_pm	heat conductivity
μ_α	phase viscosity	q^h	heat source term
D_α^κ	diffusivity of component κ in phase α	$\boldsymbol{v}_{a,\alpha}$	advective velocity
$\boldsymbol{v}_\alpha$	velocity (Darcy or free flow)		

Table 6.1: Notation list for most of the variables and indices used in DuMu$^\mathrm{x}$.

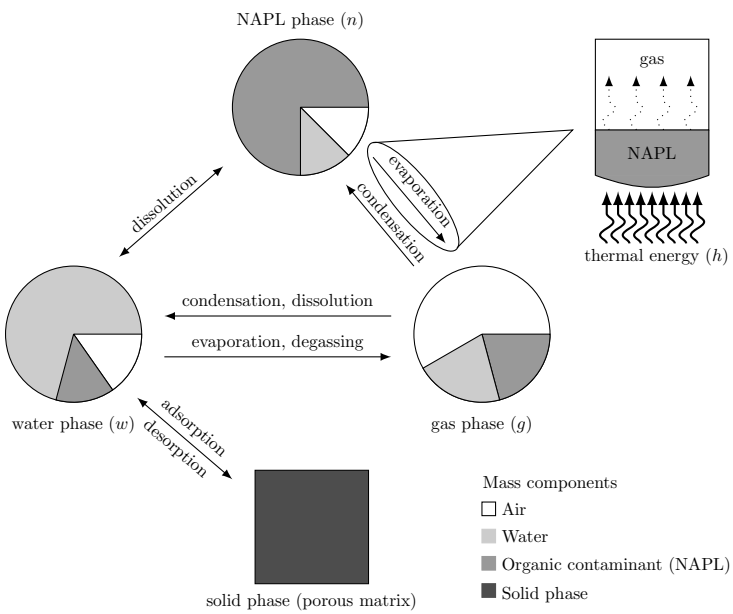


Figure 6.1: Mass and energy transfer between the phases in a water-NAPL-gas system [9]

6.1.2 Scale[1]

Depending on the scale of interest, physical and chemical processes and properties can be described using different approaches. On the molecular scale, the properties and interactions of individual molecules are described, which is only feasible for a restricted number of molecules. For larger systems, a continuum approach is used, where properties are averaged over groups of similar molecules, assuming continuous matter. This upscaling by averaging from the molecular scale results in the micro-scale, on which the system is described by the pore geometry and the distribution of distinct fluid phases within the pores. However, for larger laboratory or field-scale applications, the micro-scale is still computationally prohibitively expensive and system descriptions on the macro-scale are used for calculations. The macro-scale description is obtained by averaging over the micro-scale properties within a representative elementary volume (REV), which needs to be large enough to ensure that the averaged properties are independent of the REV size or position. However, it should in turn be much smaller than the entire domain size [12]. The detailed pore-geometry and phase-distribution information of the micro-scale is lost on the macro-scale and replaced by volume average quantities, such as porosity, permeability and phase saturation, and relations like the Darcy's law. The macro-scale is also called the REV (or Darcy) scale and is the scale of the models available in DuMu^x.

6.1.3 Porous medium properties[1]

Porosity

The porosity ϕ is defined as the fraction of the volume occupied by fluids in an REV V_{fluid} divided by the total volume of the REV V_{total}.

$$\phi = \frac{V_{\text{fluid}}}{V_{\text{total}}} = 1 - \frac{V_{\text{solid}}}{V_{\text{total}}}. \tag{6.1}$$

Intrinsic permeability

The intrinsic permeability is a measure on the REV scale of the ease of fluid flow through porous media. It relates the potential gradient and the resulting flow velocity in the Darcy equation. As the porous medium may have a structure leading to preferential flow in certain directions, intrinsic permeability is in general a tensorial quantity $\mathbf{K}$. For isotropic porous media, it can be reduced to a scalar quantity K.

[1] This subsection is taken from [13] in a slightly adapted form.

6.1.4 Mass fraction, mole fraction[1]

The composition of a phase is described by mass or mole fractions of the components. The mole fraction x_α^κ of component κ in phase α is defined as:

$$x_\alpha^\kappa = \frac{n_\alpha^\kappa}{\sum_i n_\alpha^i}, \tag{6.2}$$

where n_α^κ is the number of moles of component κ in phase α. The mass fraction X_α^κ is defined similarly using the mass of component κ in phase α instead of n_α^κ, $X_\alpha^\kappa = {}^{\text{mass}_\alpha^\kappa}/_{\text{mass}_\alpha^{\text{total}}}$. The molar mass M^κ of the component κ relates the mass fraction to the mole fraction and vice versa.

6.1.5 Fluid properties[1]

The most important fluid properties to describe fluid flow on the REV scale are density and viscosity.

Density

The density ρ_α of a fluid phase α is defined as the ratio of its mass to its volume ($\rho_\alpha = {}^{\text{mass}_\alpha}/_{\text{volume}_\alpha}$) while the molar density $\rho_{\text{mol},\alpha}$ is defined as the ratio of the number of moles per volume ($\rho_{\text{mol},\alpha} = {}^{\text{moles}_\alpha}/_{\text{volume}_\alpha}$).

Viscosity

The dynamic viscosity μ_α characterizes the resistance of a fluid to flow. As density, it is a fluid phase property. For Newtonian fluids, it relates the shear stress τ_s to the velocity gradient ${}^{dv_{\alpha,x}}/_{dy}$:

$$\tau_\text{s} = \mu_\alpha \frac{dv_{\alpha,x}}{dy}. \tag{6.3}$$

Density and viscosity are both dependent on pressure, temperature and phase composition.

6.1.6 Fluid phase interactions in porous media[1]

If more than a single fluid is present in the porous medium, the fluids interact with each other and the solids, which leads to additional properties for multi-phase systems.

Saturation

The saturation S_α of a phase α is defined as the ratio of the volume occupied by that phase to the total pore volume within an REV. As all pores are filled with some fluid, the sum of the saturations of all present phases is equal to one.

Capillary pressure

Immiscible fluids form a sharp interface as a result of differences in their intermolecular forces translating into different adhesive and cohesive forces at the fluid-fluid and fluid-fluid-solid interfaces creating interfacial tension on the microscale. From the mechanical equilibrium which has also to be satisfied

at the interface, a difference between the pressures of the fluid phases results defined as the capillary pressure p_c:

$$p_c = p_n - p_w. \tag{6.4}$$

On the microscale, p_c can be calculated from the surface tension according to the Laplace equation [see 12].

On the REV scale, however, capillary pressure needs to be defined by quantities of that scale. Several empirical relations provide expressions to link p_c to the wetting-phase saturation S_w. An example is the relation given by Brooks and Corey [7] to determine p_c based on S_e, which is the effective wetting-phase saturation, the entry pressure p_d, and the parameter λ describing the pore-size distribution:

$$p_c = p_d S_e^{-\frac{1}{\lambda}}, \tag{6.5}$$

with

$$S_e = \frac{S_w - S_{w,r}}{1 - S_{w,r}}, \tag{6.6}$$

where $S_{w,r}$ is the residual wetting phase saturation which cannot be displaced by another fluid phase and remains in the porous medium.

Relative permeability

The presence of two fluid phases in the porous medium reduces the space available for flow for each of the fluid phases. This increases the resistance to flow of the phases, which is accounted for by the means of the relative permeability $k_{r,\alpha}$, which scales the intrinsic permeability. It is a value between zero and one, depending on the saturation. The relations describing the relative permeabilities of the wetting and nonwetting phase are different as the wetting phase predominantly occupies small pores and the edges of larger pores while the nonwetting phases occupies large pores. The relative permeabilities for the wetting phase $k_{r,w}$ and the nonwetting phase are e.g. calculated as (also by Brooks and Corey [7]):

$$k_{r,w} = S_e^{\frac{2+3\lambda}{\lambda}} \tag{6.7}$$

and

$$k_{r,n} = (1 - S_e)^2 \left(1 - S_e^{\frac{2+\lambda}{\lambda}}\right). \tag{6.8}$$

6.1.7 Transport processes in porous media [1]

On the macro-scale, the transport of mass can be grouped according to the driving force of the transport process. Pressure gradients result in the advective transport of a fluid phase and all the components constituting the phase, while concentration gradients result in the diffusion of a component within a phase.

Advection

Advective transport is determined by the flow field. On the macro-scale, the velocity $\mathbf{v}$ is calculated using the Darcy equation depending on the potential gradient $(\nabla p_\alpha - \rho_\alpha \mathbf{g})$, accounting for both pressure difference and gravitation, the intrinsic permeability of the porous medium, and the viscosity μ of the fluid phase:

$$\mathbf{v} = -\frac{\mathbf{K}}{\mu}(\nabla p - \rho \mathbf{g}). \tag{6.9}$$

$\mathbf{v}$ is proportional to $(\nabla p - \rho \mathbf{g})$ with the proportional factor $\mathbf{K}/\mu$. This equation can be extended to calculate the velocity $\mathbf{v}_\alpha$ of phase α in the case of two-phase flow by considering the relative permeability $k_{\mathrm{r},\alpha}$ (Section 6.1.6):

$$\mathbf{v}_\alpha = -\frac{k_{\mathrm{r},\alpha}\mathbf{K}}{\mu_\alpha}(\nabla p_\alpha - \rho_\alpha \mathbf{g}) \tag{6.10}$$

Diffusion

Molecular diffusion is a process determined by the concentration gradient. It is commonly modeled as Fickian diffusion following Fick's first law:

$$\mathbf{j_d} = -\rho_\alpha D_\alpha^\kappa \nabla X_\alpha^\kappa, \tag{6.11}$$

where D_α^κ is the molecular diffusion coefficient of component κ in phase α. In a porous medium, the actual path lines are tortuous due to the impact of the solid matrix. This tortuosity and the impact of the presence of multiple fluid phases is accounted for by using an effective diffusion coefficient $D_{\mathrm{pm},\alpha}^\kappa$:

$$D_{\mathrm{pm},\alpha}^\kappa = \phi \tau_\alpha S_\alpha D_\alpha^\kappa, \tag{6.12}$$

where τ_α is the tortuosity of phase α.

6.1.8 Gas mixing laws

Prediction of the $p - \varrho - T$ behavior of gas mixtures is typically based on two (contradicting) concepts: Dalton's law or Amagat's law. In the following the two concepts will be explained in more detail.

Dalton's law

Dalton's law assumes that the gases in the mixture are non-interacting (with each other) and each gas independently applies its own pressure (partial pressure), the sum of which is the total pressure:

$$p = \sum_i p_i. \tag{6.13}$$

Here p_i refers to the partial pressure of component i. As an example, if two equal volumes of gas A and gas B are mixed, the volume of the mixture stays the same but the pressures add up (see Figure 6.2). The density of the mixture, ϱ, can be calculated as follows:

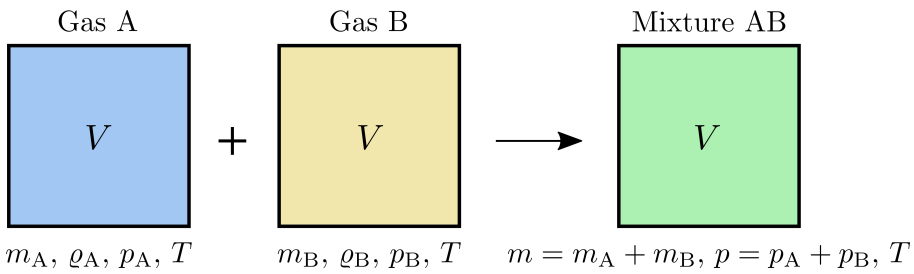


Figure 6.2: Dalton's law visualized

$$\varrho = \frac{m}{V} = \frac{m_A + m_B}{V} = \frac{\varrho_A V + \varrho_B V}{V} = \varrho_A + \varrho_B, \tag{6.14}$$

or for an arbitrary number of gases:

$$\varrho = \sum_i \varrho_i; \quad \varrho_m = \sum_i \varrho_{m,i}. \tag{6.15}$$

Amagat's law

Amagat's law assumes that the volumes of the component gases are additive; the interactions of the different gases are the same as the average interactions of the components. This is known as Amagat's law:

$$V = \sum_i V_i. \tag{6.16}$$

As an example, if two volumes of gas A and B at equal pressure are mixed, the pressure of the mixture stays the same, but the volumes add up (see Figure 6.3). The density of the mixture, ϱ, can be

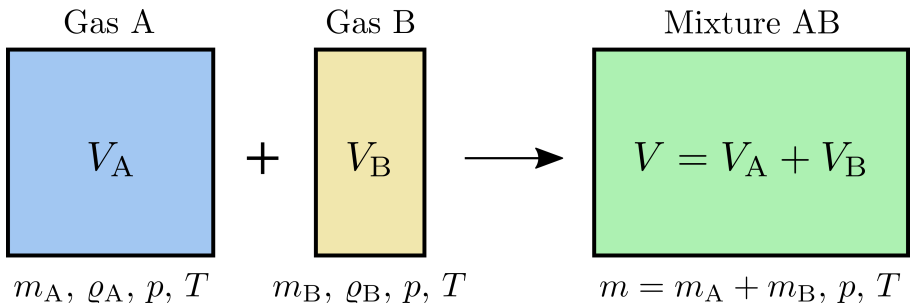


Figure 6.3: Amagat's law visualized

calculated as follows:

$$\varrho = \frac{m}{V} = \frac{m}{V_A + V_B} = \frac{m}{\frac{m_A}{\varrho_A} + \frac{m_B}{\varrho_B}} = \frac{m}{\frac{X_A m}{\varrho_A} + \frac{X_B m}{\varrho_B}} = \frac{1}{\frac{X_A}{\varrho_A} + \frac{X_B}{\varrho_B}}, \tag{6.17}$$

or for an arbitrary number of gases:

$$\varrho = \frac{1}{\sum_i \frac{X_i}{\varrho_i}}; \quad \varrho_m = \frac{1}{\sum_i \frac{x_i}{\varrho_{m,i}}}. \tag{6.18}$$

Ideal gases

An ideal gas is defined as a gas whose molecules are spaced so far apart that the behavior of a molecule is not influenced by the presence of other molecules. This assumption is usually valid at low pressures and high temperatures. The ideal gas law states that, for one gas:

$$p = \varrho \frac{RT}{M}; \quad p = \varrho_m RT. \tag{6.19}$$

Using the assumption of ideal gases and either Dalton's law or Amagat's law lead to the density of the mixture, ϱ, as:

$$\varrho = \frac{p}{RT} \sum_i M_i x_i; \quad \varrho_m = \frac{p}{RT}. \tag{6.20}$$

6.1.9 Available Models

A list of all available models can be found in the Doxygen documentation at `https://dumux.org/docs/doxygen/releases/3.5/modules.html`. The documentation includes a detailed description for every model.

6.2 Temporal Discretization and Solution Strategies

In this section, the temporal discretization as well as solution strategies (monolithic/sequential) are presented.

6.2.1 Temporal discretization

Our systems of partial differential equations are discretized in space and in time.

Let us consider the general case of a balance equation of the following form

$$\frac{\partial m(u)}{\partial t} + \nabla \cdot \mathbf{f}(u, \nabla u) + q(u) = 0, \tag{6.21}$$

seeking an unknown quantity u in terms of storage m, flux $\mathbf{f}$ and source q. All available Dumux models can be written mathematically in form of (6.21) with possibly vector-valued quantities u, m, q and a tensor-valued flux $\mathbf{f}$. For the sake of simplicity, we assume scalar quantities u, m, q and a vector-valued flux $\mathbf{f}$ in the notation below.

For discretizing (6.21), we need to choose an approximation for the temporal derivative $\partial m(u)/\partial t$. While many elaborate methods for this approximation exist, we focus on the simplest one of a first order difference quotient

$$\frac{\partial m(u_{k/k+1})}{\partial t} \approx \frac{m(u_{k+1}) - m(u_k)}{\Delta t_{k+1}} \tag{6.22}$$

for approximating the solution u at time t_k (forward) or t_{k+1} (backward). The question of whether to choose the forward or the backward quotient leads to the explicit and implicit Euler method, respectively. In case of the former, inserting (6.22) in (6.21) at time t_k leads to

$$\frac{m(u_{k+1}) - m(u_k)}{\Delta t_{k+1}} + \nabla \cdot \mathbf{f}(u_k, \nabla u_k) + q(u_k) = 0, \tag{6.23}$$

whereas the implicit Euler method is described as

$$\frac{m(u_{k+1}) - m(u_k)}{\Delta t_{k+1}} + \nabla \cdot \mathbf{f}(u_{k+1}, \nabla u_{k+1}) + q(u_{k+1}) = 0. \tag{6.24}$$

Once the solution u_k at time t_k is known, it is straightforward to determine $m(u_{k+1})$ from (6.23), while attempting to do the same based on (6.24) involves the solution of a system of equations. On the other hand, the explicit method (6.23) is stable only if the time step size Δt_{k+1} is below a certain limit that depends on the specific balance equation, whereas the implicit method (6.24) is unconditionally stable.

6.2.2 Solution strategies to solve equations

The governing equations of each model can be solved monolithically or sequentially. The basic idea of the sequential algorithm is to reformulate the equations of multi-phase flow into one equation for pressure and equations for phase/component/... transport. The pressure equation is the sum of the mass balance equations and thus considers the total flow of the fluid system. The new set of equations is considered as decoupled (or weakly coupled) and can thus be solved sequentially. The most popular sequential model is the fractional flow formulation for two-phase flow which is usually implemented applying an IMplicit Pressure Explicit Saturation algorithm (IMPES). In comparison to solving the equations monolithically, the sequential structure allows the use of different discretization methods for the different equations. The standard method used in the sequential algorithm is a cell-centered finite volume method. Further schemes, so far only available for the two-phase pressure equation, are cell-centered finite volumes with multi-point flux approximation (Mpfa-O method) and mimetic finite differences. An h-adaptive implementation of both sequential algorithms is provided for two dimensions.

6.3 Spatial Discretization

We discretize space with cell-centered finite volume methods (6.3.1), the box method (6.3.2) or a staggered grid scheme (6.3.3). Grid adaption is available for both box and cell-centered finite volume method. In general, the spatial parameters, especially the porosity, have to be assigned on the coarsest level of discretization.

6.3.1 Cell Centered Finite Volume Methods – A Short Introduction

Cell-centered finite volume methods use the elements of the grid as control volumes. For each control volume the discrete values are determined at the element/control volume center (not required to be the barycenters).

We consider a domain $\Omega \subset \mathbb{R}^d$, $d \in \{2, 3\}$ with boundary $\Gamma = \partial\Omega$. Within this section, we consider the following elliptic problem

$$\begin{aligned} \nabla \cdot (-\mathbf{\Lambda} \nabla u) &= q && \text{in}\,\Omega \\ (-\mathbf{\Lambda} \nabla u) \cdot \mathbf{n} &= v_N && \text{on}\,\Gamma_N \\ u &= u_D && \text{on}\,\Gamma_D. \end{aligned} \tag{6.25}$$

Here, $\mathbf{\Lambda} = \mathbf{\Lambda}(\mathbf{x}, \mathbf{u})$ is a symmetric and positive definite tensor of second rank (e.g. permeability, diffusivity, etc.), $u = u(\mathbf{x})$ is unknown and $q = q(\mathbf{x}, \mathbf{u})$ is a source/sink. We denote by $\mathcal{M}$ the mesh

that results from the division of the domain Ω into n_e control volumes $K \subset \Omega$. Each K is a polygonal open set such that $K \cap L = \emptyset, \forall K \neq L$ and $\overline{\Omega} = \cup_{K \in \mathcal{M}} \overline{K}$.

For the derivation of the finite-volume formulation we integrate the first equation of (6.25) over a control volume K and apply the Gauss divergence theorem:

$$\int_{\partial K} (-\mathbf{\Lambda} \nabla u) \cdot \mathbf{n} \, \mathrm{d}\Gamma = \int_K q \, \mathrm{d}x. \tag{6.26}$$

Splitting the control volume boundary ∂K into a finite number of faces $\sigma \subset \partial K$ (such that $\sigma = \overline{K} \cap \overline{L}$ for some neighboring control volume L) and replacing the exact fluxes by an approximation, i.e. $F_{K,\sigma} \approx \int_\sigma (-\mathbf{\Lambda}_K \nabla u) \cdot \mathbf{n} \mathrm{d}\Gamma$ (here $\mathbf{\Lambda}_K$ is the value of $\mathbf{\Lambda}$ associated with control volume K), yield

$$\sum_{\sigma \subset \partial K} F_{K,\sigma} = Q_K, \quad \forall \, K \in \mathcal{M}, \tag{6.27}$$

where $F_{K,\sigma}$ is the discrete flux through face σ flowing out of cell K and $Q_K := \int_K q \, \mathrm{d}x$ is the integrated source/sink term. Equation (6.27) is the typical cell-centered finite-volume formulation. Finite-volume schemes differ in the way how the term $(\mathbf{\Lambda}_K \nabla u) \cdot \mathbf{n}$ is approximated (i.e. the choice of the fluxes $F_{K,\sigma}$). Using the symmetry of the tensor $\mathbf{\Lambda}_K$, this term can be rewritten as $\nabla u \cdot \mathbf{\Lambda}_K \mathbf{n}$, which corresponds to the directional derivative of u in co-normal direction $\mathbf{\Lambda}_K \mathbf{n}$. In the following, the main ideas of the two-point flux approximation and the multi-point flux approximation methods are briefly described. Hereby, we restrict the discussion to the two-dimensional case.

Please also note that other types of equations, e.g. instationary parabolic problems, can be discretized by applying some time discretization scheme to the time derivatives and by using the finite-volume scheme for the flux discretization. For simplicity the discussion is restricted to the elliptic problem (6.25).

Tpfa Method

The linear two-point flux approximation is a simple but robust cell-centered finite-volume scheme, which is commonly used in commercial software. This scheme can be derived by using the co-normal decomposition, which reads

$$\mathbf{\Lambda}_K \mathbf{n}_{K,\sigma} = t_{K,\sigma} \mathbf{d}_{K,\sigma} + \mathbf{d}^\perp_{K,\sigma}, \quad t_{K,\sigma} = \frac{\mathbf{n}^T_{K,\sigma} \mathbf{\Lambda}_K \mathbf{d}_{K,\sigma}}{\mathbf{d}^T_{K,\sigma} \mathbf{d}_{K,\sigma}}, \; \mathbf{d}^\perp_{K,\sigma} = \mathbf{\Lambda}_K \mathbf{n}_{K,\sigma} - t_{K,\sigma} \mathbf{d}_{K,\sigma}, \tag{6.28}$$

with the tensor $\mathbf{\Lambda}_K$ associated with control volume K, the distance vector $\mathbf{d}_{K,\sigma} := \mathbf{x}_\sigma - \mathbf{x}_K$ and $\mathbf{d}^T_{K,\sigma} \mathbf{d}^\perp_{K,\sigma} = 0$, see Figure 6.4 for the used notations. The same can be done for the conormal $\mathbf{\Lambda}_L \mathbf{n}_{L,\sigma}$. The $t_{K,\sigma}$ and $t_{L,\sigma}$ are the transmissibilities associated with the face σ. These transmissibilities are calculated in DuMu^x by using the function `computeTpfaTransmissibility`.

With these notations, it follows that for each cell K and face σ

$$\nabla u \cdot \mathbf{\Lambda}_K \mathbf{n}_{K,\sigma} = t_{K,\sigma} \nabla u \cdot \mathbf{d}_{K,\sigma} + \nabla u \cdot \mathbf{d}^\perp_{K,\sigma}. \tag{6.29}$$

For the Tpfa scheme, the second part in the above equation is neglected. By using the fact that $\nabla u \cdot \mathbf{d}_{K,\sigma} \approx u_\sigma - u_K$, the discrete fluxes for face σ are given by

$$F_{K,\sigma} = -|\sigma| t_{K,\sigma} (u_\sigma - u_K), \qquad F_{L,\sigma} = -|\sigma| t_{L,\sigma} (u_\sigma - u_L). \tag{6.30}$$

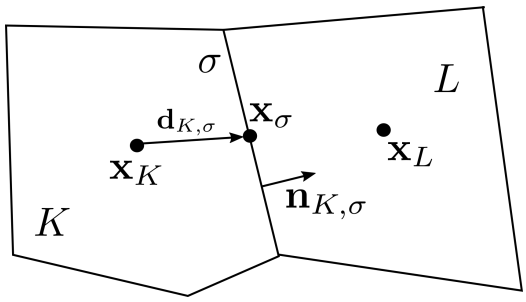


Figure 6.4: Two neighboring control volumes sharing the face σ.

Enforcing local flux conservation, i.e. $F_{K,\sigma} + F_{L,\sigma} = 0$, results in

$$u_\sigma = \frac{t_{K,\sigma} u_K + t_{L,\sigma} u_L}{t_{K,\sigma} + t_{L,\sigma}}. \tag{6.31}$$

With this, the fluxes (6.30) are rewritten as

$$F_{K,\sigma} = |\sigma| \frac{t_{K,\sigma} t_{L,\sigma}}{t_{K,\sigma} + t_{L,\sigma}} (u_K - u_L), \quad F_{L,\sigma} = |\sigma| \frac{t_{K,\sigma} t_{L,\sigma}}{t_{K,\sigma} + t_{L,\sigma}} (u_L - u_K). \tag{6.32}$$

By neglecting the orthogonal term, the consistency of the scheme is lost for general grids, where $\nabla u \cdot \mathbf{d}_{K,\sigma}^{\perp} \neq 0$. The consistency is achieved only for so-called K-orthogonal grids for which $\mathbf{d}_{K,\sigma}^{\perp} = 0$. For such grids we deduce that

$$\frac{t_{K,\sigma} t_{L,\sigma}}{t_{K,\sigma} + t_{L,\sigma}} = \frac{\tau_{K,\sigma} \tau_{L,\sigma}}{\tau_{K,\sigma} d_{L,\sigma} + \tau_{L,\sigma} d_{K,\sigma}}, \tag{6.33}$$

with $\tau_{K,\sigma} := \mathbf{n}_{K,\sigma} \mathbf{\Lambda}_K \mathbf{n}_{K,\sigma}, \tau_{L,\sigma} := \mathbf{n}_{L,\sigma} \mathbf{\Lambda}_L \mathbf{n}_{L,\sigma}$, $d_{K,\sigma} := \mathbf{n}_{K,\sigma} \cdot \mathbf{d}_{K,\sigma}$, and $d_{L,\sigma} := \mathbf{n}_{L,\sigma} \cdot \mathbf{d}_{L,\sigma}$. This reduces, for the case of scalar permeability, to a distance weighted harmonic averaging of permeabilities.

Mpfa Method

Expressions for the face fluxes $F_{K,\sigma}$ are obtained by introducing intermediate face unknowns u_σ in addition to the cell unknowns u_K and enforcing the physically motivated continuity of fluxes and continuity of the solution across the faces. For a face σ between the two polygons K and L these conditions read:

$$\begin{aligned} F_{K,\sigma} + F_{L,\sigma} &= 0 \\ u_{K,\sigma} = u_{L,\sigma} &= u_\sigma. \end{aligned} \tag{6.34}$$

Using these conditions, the intermediate face unknowns u_σ can be eliminated and the fluxes are expressed as a function of the cell unknowns u_N and associated transmissibilities $t_{K,\sigma}^N$:

$$F_{K,\sigma} = \sum_{N \in \mathcal{S}_{K,\sigma}} t_{K,\sigma}^N u_N. \tag{6.35}$$

The main difference between the various finite-volume schemes available is the assembly of the face fluxes, i.e. the computation of the $t_{K,\sigma}^N$ and the size of $\mathcal{S}_{K,\sigma}$. For the Tpfa, that has been presented in

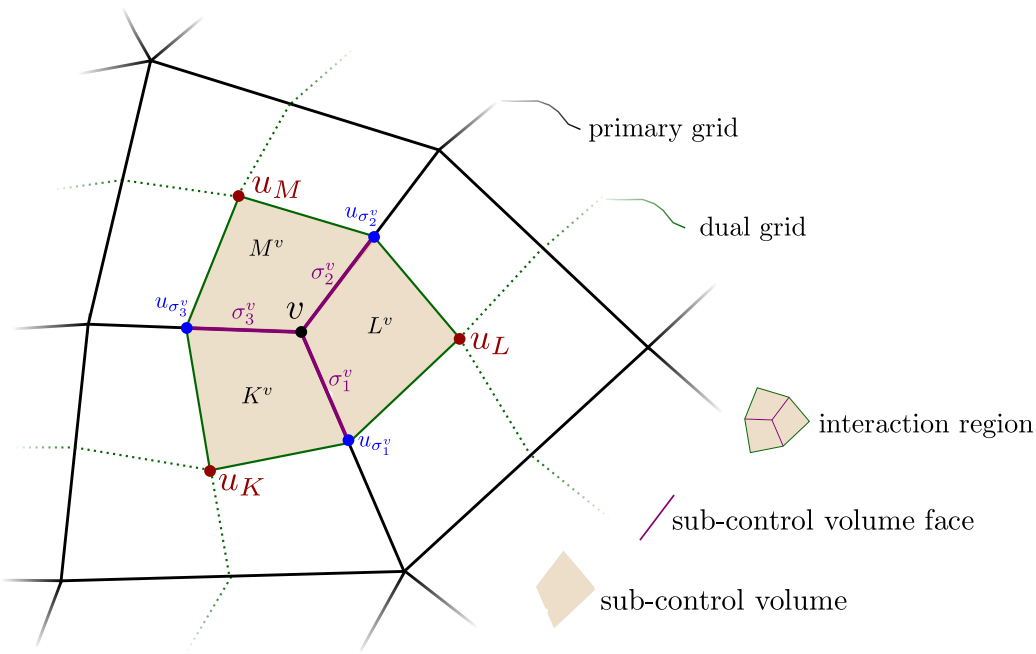


Figure 6.5: Interaction region for the Mpfa-O method. The graphic on the right illustrates how the sub-control volume L^v and face σ_2^v are embedded in cell L. Note that the face stencils for all sub-control volume faces in the depicted interaction region are $\mathcal{S}_{\sigma_i^v} = \{K, L, M\}$, meaning that the fluxes over the sub-control volume faces depend on the three cell unknowns u_K, u_L, u_M.

the last section, the stencil and transmissibilities are given as

$$\mathcal{S}_{K,\sigma} = \{K, L\}, \quad t_{K,\sigma}^K = |\sigma| \frac{t_{K,\sigma} t_{L,\sigma}}{t_{K,\sigma} + t_{L,\sigma}}, \; t_{K,\sigma}^L = -|\sigma| \frac{t_{K,\sigma} t_{L,\sigma}}{t_{K,\sigma} + t_{L,\sigma}},$$

with $t_{K,\sigma}, t_{L,\sigma}$ as defined in equation (6.28).

In the following, a multi-point flux approximation method (Mpfa-O method), which was introduced in Aavatsmark [1], is presented. The main difference to the Tpfa scheme is the fact that a consistent discrete gradient is constructed, i.e. the term $\nabla u \cdot \mathbf{d}_{K,\sigma}^{\perp}$ is not neglected.

For this scheme, a dual grid is created by connecting the barycenters of the cells with the barycenters of the faces ($d = 2$) or the barycenters of the faces and edges ($d = 3$). This divides each cell into sub-control volumes K^v. Analogously, each face is sub-divided into sub-control volume faces σ^v, see Figure 6.5. We allow for piecewise constant $\mathbf{\Lambda}$ (denoted as $\mathbf{\Lambda}_K$ for each cell K) and construct discrete gradients $\nabla_{\mathcal{D}}^{K^v} u$ (per sub-control volume K^v). In the following, we restrict our discussion to the two-dimensional setup that is shown in Figure 6.5. Here, the discrete gradients are constructed to be consistent such that the following conditions hold:

$$\nabla_{\mathcal{D}}^{K^v} u \cdot (\mathbf{x}_{\sigma_1^v} - \mathbf{x}_K) = u_{\sigma_1^v} - u_K, \quad \nabla_{\mathcal{D}}^{K^v} u \cdot (\mathbf{x}_{\sigma_3^v} - \mathbf{x}_K) = u_{\sigma_3^v} - u_K. \tag{6.36}$$

Thus, a discrete gradient (for sub-control volume K^v) that fulfills these conditions is given as

$$\nabla_{\mathcal{D}}^{K^v} u = \mathbb{D}_{K^v}^{-T} \begin{bmatrix} u_{\sigma_1^v} - u_K \\ u_{\sigma_3^v} - u_K \end{bmatrix}, \qquad \text{with } \mathbb{D}_{K^v} := \begin{bmatrix} \mathbf{x}_{\sigma_1^v} - \mathbf{x}_K & \mathbf{x}_{\sigma_3^v} - \mathbf{x}_K \end{bmatrix}. \tag{6.37}$$

This enables us to write the discrete flux across σ_1^v from cell K as follows:

$$F_{K,\sigma_1^v} := -|\sigma_1^v| \mathbf{n}_{\sigma_1^v}^T \mathbf{\Lambda}_K \nabla_{\mathcal{D}}^{K^v} u. \tag{6.38}$$

Inserting the discrete gradient, yields

$$F_{K,\sigma_1^v} = \omega_{K,\sigma_1^v\sigma_1^v}(u_K - u_{\sigma_1^v}) + \omega_{K,\sigma_1^v\sigma_3^v}(u_K - u_{\sigma_3^v}), \tag{6.39}$$

with $(\omega_{K,\sigma_1^v\sigma_1^v}, \omega_{K,\sigma_1^v\sigma_3^v})^T = |\sigma_1^v| \mathbb{D}_{K^v}^{-1} \mathbf{\Lambda}_K \mathbf{n}_{\sigma_1^v}$. These values are calculated in DuMu^{x} by using the function `computeMpfaTransmissibility`.

To deduce a cell-centered scheme, the introduced face unknowns $u_{\sigma_i^v}$ have to be eliminated. This is done by enforcing flux continuity for each sub-control volume face, i.e.

$$F_{K,\sigma_1^v} + F_{L,\sigma_1^v} = 0, \tag{6.40}$$

$$F_{K,\sigma_3^v} + F_{M,\sigma_3^v} = 0, \tag{6.41}$$

$$F_{L,\sigma_2^v} + F_{M,\sigma_2^v} = 0. \tag{6.42}$$

This results in a system of equations for the face unknowns $\mathbf{u}_\sigma$

$$\mathbb{A}^{3\times 3}\mathbf{u}_\sigma = \mathbb{B}^{3\times 3}\mathbf{u}, \tag{6.43}$$

where $\mathbf{u}$ contains the three cell unknowns u_K, u_L, u_M and $\mathbf{u}_\sigma$ the three face unknowns $u_{\sigma_1^v}, u_{\sigma_2^v}, u_{\sigma_3^v}$. Inserting these face unknowns into the flux expression (6.39) yields

$$F_{K,\sigma_i^v} = \sum_{N\in\{K,L,M\}} t_{K,\sigma_i^v}^N u_N = \mathbf{t}_{K,\sigma_i^v} \cdot \mathbf{u}, \tag{6.44}$$

for each cell K and sub-control volume face σ_i^v.

6.3.2 Box Method – A Short Introduction

The so called box method unites the advantages of the finite-volume (FV) and finite-element (FE) methods.

First, the model domain Ω is discretized with a FE mesh consisting of nodes i and corresponding elements E_k. Then, a secondary FV mesh is constructed by connecting the midpoints and barycenters of the elements surrounding node i creating a box B_i around node i (see Figure 6.6a).

The FE mesh divides the box B_i into subcontrolvolumes (scv's) b_i^k (see Figure 6.6b). Figure 6.6c shows the finite element E_k and the scv's b_i^k inside E_k, which belong to four different boxes B_i. Also necessary for the discretization are the faces of the subcontrolvolumes (scvf's) e_{ij}^k between the scv's b_i^k and b_j^k, where $|e_{ij}^k|$ is the length of the scvf. The integration points x_{ij}^k on e_{ij}^k and the outer normal vector $\mathbf{n}_{ij}^k$ are also to be defined (see Figure 6.6c).

The advantage of the FE method is that unstructured grids can be used, while the FV method is mass conservative. The idea is to apply the FV method (balance of fluxes across the interfaces) to each FV box B_i and to get the fluxes across the interfaces e_{ij}^k at the integration points x_{ij}^k from the FE approach. Consequently, at each scvf the following expression results:

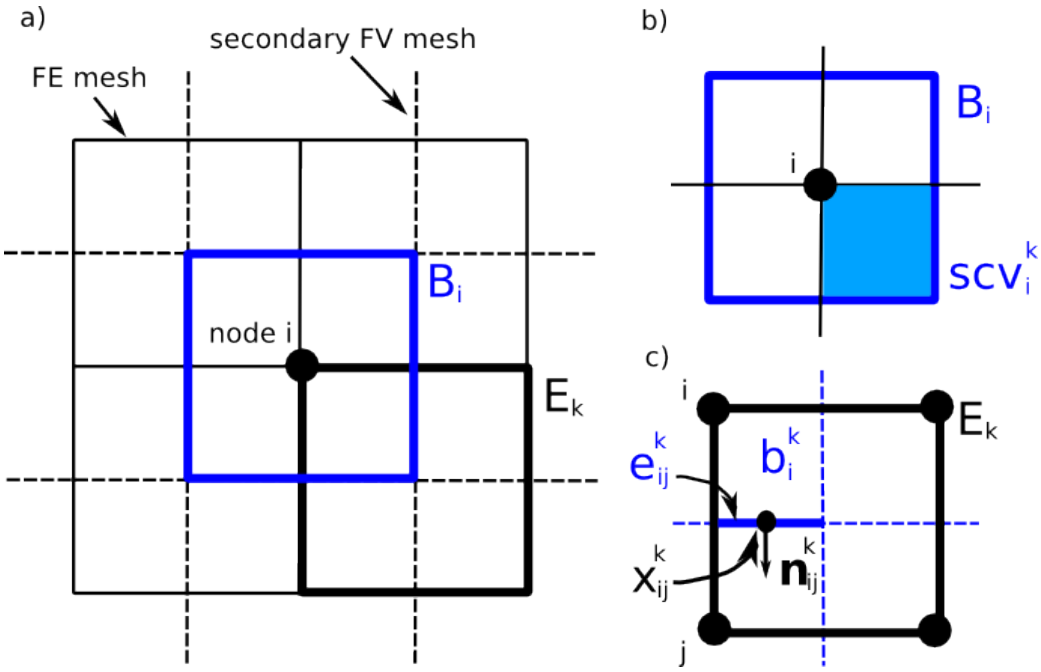


Figure 6.6: Discretization of the box method

$$f(\tilde{u}(x_{ij}^k)) \cdot \mathbf{n}_{ij}^k \, |e_{ij}^k| \qquad \text{with} \qquad \tilde{u}(x_{ij}^k) = \sum_i N_i(x_{ij}^k) \cdot \hat{u}_i. \tag{6.45}$$

In the following, the discretization of the balance equation is going to be derived. From the Reynolds transport theorem follows the general balance equation:

$$\underbrace{\int_\Omega \frac{\partial}{\partial t} u \, \mathrm{d}x}_{1} + \underbrace{\int_{\partial\Omega} (\mathbf{v}u + \mathbf{w}) \cdot \mathbf{n} \, \mathrm{d}\Gamma}_{2} = \underbrace{\int_\Omega q \, \mathrm{d}x}_{3} \tag{6.46}$$

$$f(u) = \int_\Omega \frac{\partial u}{\partial t} \, \mathrm{d}x + \int_\Omega \nabla \cdot \underbrace{[\mathbf{v}u + \mathbf{w}(u)]}_{F(u)} \, \mathrm{d}x - \int_\Omega q \, \mathrm{d}x = 0 \tag{6.47}$$

where term 1 describes the changes of entity u within a control volume over time, term 2 the advective, diffusive and dispersive fluxes over the interfaces of the control volume and term 3 is the source and sink term. Ω denotes the model domain and $F(u) = F(\mathbf{v}, p) = F(\mathbf{v}(x,t), p(x,t))$.

Like the FE method, the box method follows the principle of weighted residuals. In the function $f(u)$ the unknown u is approximated by discrete values at the nodes of the FE mesh $\hat{u}_i$ and linear basis functions N_i yielding an approximate function $f(\tilde{u})$. For $u \in \{\mathbf{v}, p, x^\kappa\}$, this means:

$$\tilde{p} = \sum_i N_i \hat{p}_i \tag{6.48}$$

$$\nabla\tilde{p} = \sum_i \nabla N_i \hat{p}_i \tag{6.51}$$

$$\tilde{\mathbf{v}} = \sum_i N_i \hat{\mathbf{v}}_i \tag{6.49}$$

$$\nabla\tilde{\mathbf{v}} = \sum_i \nabla N_i \hat{\mathbf{v}}_i \tag{6.52}$$

$$\tilde{x}^\kappa = \sum_i N_i \hat{x}_i^\kappa \tag{6.50}$$

$$\nabla\tilde{x}^\kappa = \sum_i \nabla N_i \hat{x}_i^\kappa. \tag{6.53}$$

Due to the approximation with node values and basis functions, the differential equations are not exactly fulfilled anymore but a residual ε is produced.

$$f(u) = 0 \qquad \Rightarrow \qquad f(\tilde{u}) = \varepsilon \tag{6.54}$$

Application of the principle of weighted residuals, meaning the multiplication of the residual ε with a weighting function W_j and claiming that this product has to vanish within the whole domain,

$$\int_\Omega \varepsilon W_j \,\mathrm{d}x \overset{!}{=} 0 \qquad \text{with} \qquad \sum_j W_j = 1 \tag{6.55}$$

yields the following equation:

$$\int_\Omega \frac{\partial\tilde{u}}{\partial t} W_j \,\mathrm{d}x + \int_\Omega \left[\nabla\cdot F(\tilde{u})\right] W_j \,\mathrm{d}x - \int_\Omega q W_j \,\mathrm{d}x = \int_\Omega \varepsilon W_j \,\mathrm{d}x \overset{!}{=} 0. \tag{6.56}$$

For standard Galerkin schemes, the weighting functions W_j are chosen the same as the ansatz functions N_j. However, this does not yield a locally mass-conservative scheme. Therefore, for the Box method, the weighting functions W_j are chosen as the piece-wise constant functions over a control volume box B_j, i.e.

$$W_j(x) = \begin{cases} 1 & x \in B_j \\ 0 & x \notin B_j. \end{cases} \tag{6.57}$$

Thus, the Box method is a Petrov-Galerkin scheme, where the weighting functions do not belong to the same function space as the ansatz functions.

Inserting definition (6.57) into equation (6.56) and using the Green-Gaussian integral theorem results in

$$\int_{B_j} \frac{\partial\tilde{u}}{\partial t} \,\mathrm{d}x + \int_{\partial B_j} F(\tilde{u})\cdot\mathbf{n}\,\mathrm{d}\Gamma_{B_j} - \int_{B_j} q \,\mathrm{d}x \overset{!}{=} 0, \tag{6.58}$$

which has to hold for every box B_j.

The first term in equation (6.58) can be written as

$$\int_{B_j} \frac{\partial\tilde{u}}{\partial t} \,\mathrm{d}x = \frac{d}{dt}\int_{B_j} \sum_i \hat{u}_i N_i \,\mathrm{d}x = \sum_i \frac{\partial\hat{u}_i}{\partial t} \int_{B_j} N_i \,\mathrm{d}x. \tag{6.59}$$

Here, a mass lumping technique is applied by assuming that the storage capacity is reduced to the nodes. This means that the integrals $M_{i,j} = \int_{B_j} N_i \,\mathrm{d}x$ are replaced by some mass lumped terms $M_{i,j}^{lump}$ which are defined as

$$M_{i,j}^{lump} = \begin{cases} |B_j| & j = i \\ 0 & j \neq i, \end{cases} \tag{6.60}$$

where $|B_j|$ is the volume of the FV box B_j associated with node j. The application of this assumption yields

$$|B_j|\frac{\partial \hat{u}_j}{\partial t} + \int_{\partial B_j} F(\tilde{u}) \cdot \mathbf{n} \, \mathrm{d}\Gamma_{B_j} - Q_j = 0, \tag{6.61}$$

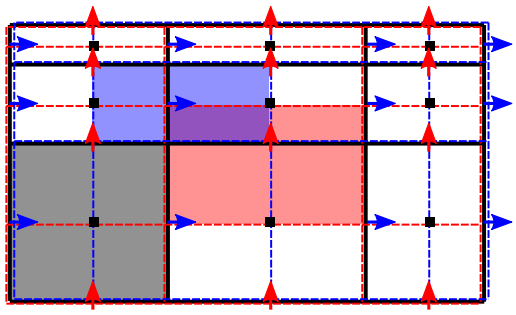
where Q_j is an approximation (using some quadrature rule) of the integrated source/sink term $\int_{B_j} q \, \mathrm{d}x$.

Using an implicit Euler time discretization finally leads to the discretized form which will be applied to the mathematical flow and transport equations:

$$|B_j|\frac{\hat{u}_j^{n+1} - \hat{u}_j^n}{\Delta t} + \int_{\partial B_j} F(\tilde{u}^{n+1}) \cdot \mathbf{n} \, \mathrm{d}\Gamma_{B_j} - Q_j^{n+1} = 0. \tag{6.62}$$

Equation (6.62) has to be fulfilled for each box B_j.

6.3.3 Staggered Grid – A Short Introduction



- ■ Cell-centered primary variables ($p_\alpha, x_\alpha^\kappa, T, k, \epsilon ...$)
- $v_{\alpha,x}$
- $v_{\alpha,y}$
- finite volume mesh and boundaries of the control volumes centered around the scalar variables
- boundaries of the staggered control volumes located around the velocity x-component
- boundaries of the staggered control volumes located around the velocity y-component

Figure 6.7: Discretization of the staggered-grid method. The figure shows the different control volume arrangements, which are staggered with respect to each other. There are the control volumes centered around the scalar primary variables in black, the control volumes located around the x-component of the velocity in blue and the control volumes located around the y-components of the velocity in red. The control volume boundaries are given by lines. Additionally, there is one shaded example control volume each.
In the two-dimensional free-flow models, the continuity equation is discretized using the black control volumes, the x-component of the momentum equation is discretized using the blue control volumes and the y-component is discretized using the red control volumes. In three dimensions this works analogously.

The staggered-grid or marker-and-cell method uses a finite volume method with different control volumes for different equations. There are control volumes centered around the scalar primary variables. They correspond to the finite volume mesh. Additionally, there are control volumes located around the x, y and (in 3D) z velocity components which are shifted in the x, y and z direction, such that the velocity components are located on the edges of the cell-centered finite volume mesh (see Figure 6.7). As for the cell-centered method, the fluxes are evaluated at the edges of each control volume with a two-point flux approximation, cf. 6.3.1.

The staggered-grid method is robust, mass conservative, and free of pressure oscillations but should, as the cell-centered TPFA method, only be applied for structured grids. Currently, all free-flow models in DuMu^x use the staggered-grid discretization.

6.4 Steps of a DuMu^x Simulation

This chapter is supposed to give a short overview over how things are "handed around" in DuMu^x. It is not a comprehenisve guide through the modeling framework of DuMu^x, but hopefully it will help getting to grips with it.

In Section 6.4.1 the structure of DuMu^x is shown from a *content* point of view.

6.4.1 Structure – by Content

In Figure 6.8, the algorithmic representations of a monolithical solution scheme is illustrated down to the element level.

```
1. main   2. time step   3. Newton   4. element

initialize
foreach time step
        foreach NEWTON iteration
                foreach element
                        - calculate element
                          residual vector and
                          Jacobian matrix
                        - assemble into global
                          residual vector and
                          Jacobian matrix
                endfor
                solve linear system
                update solution
                check for NEWTON convergence
        endfor
        - adapt time step size,
          possibly redo with smaller step size
        - write result
endfor
finalize
```

Figure 6.8: Structure of a monolithical solution scheme in DuMu^x.

6.4.2 Structure – by Implementation

A possible starting point to understand how the above mentioned algorithm is implemented within DuMu^x, is the example main file `https://git.iws.uni-stuttgart.de/dumux-repositories/dumux-course/-/blob/releases/3.5/exercises/exercise-mainfile/exercise1pamain.cc`

6.5 Property System

A high level overview over the property system's design and principle ideas are given, then follows a reference and a self-contained example.

6.5.1 Motivation and features

The DuMu^x property system is a traits system which allows easy inheritance. In the context of the DuMu^x property system, a property is an arbitrary class body which may contain type definitions, values and methods. Just like normal classes, properties can be arranged in hierarchies. In the context of the DuMu^x property system, nodes of the inheritance hierarchy are called *type tags*.

It also supports *property nesting*. Property nesting means that the definition of a property can depend on the value of other properties which may be defined for arbitrary levels of the inheritance hierarchy.

6.5.2 How-to

All source files which use the property system should include the header file `dumux/common/properties/propertysystem.hh`. Declaration of type tags and property tags as well as defining properties must be done inside the namespace `Dumux::Properties`.

Defining Type Tags

New nodes in the type tag hierarchy can be defined in the `TTag` namespace using

```
// Create new type tags
namespace TTag {
struct NewTypeTagName { using InheritsFrom = std::tuple<BaseTagName1, BaseTagName2, ...>; };
} // end namespace TTag
```

where the `InheritsFrom` alias is optional. To avoid inconsistencies in the hierarchy, each type tag may be defined only once for a program. If you call `GetProp` the property system will first look for the properties defined in `BaseTagName1` in the `InheritsFrom` list. If a defined property is found this property is returned. If no defined property is found the search will continue in the ancestors of `BaseTagName1`. If again no defined property is found the search will continue in the second `BaseTagName2` in the list, and so on. If no defined property is found at all, a compiler error is triggered.

Example:

```
namespace Dumux {
namespace Properties {
namespace TTag {
struct MyBaseTypeTag1 {};
struct MyBaseTypeTag2 {};

```

```
struct MyDerivedTypeTag { using InheritsFrom = std::tuple<MyBaseTypeTag1, MyBaseTypeTag2>; };
} // end namespace TTag
}}
```

Defining new Property Tags

New property tags are defined using

```
template<class TypeTag, class MyTypeTag>
struct NewPropTagName { using type = UndefinedProperty; };
```

Example:

```
namespace Dumux {
namespace Properties {
template<class TypeTag, class MyTypeTag>
struct MyPropertyTag { using type = UndefinedProperty; };
}}
```

If you need to forward declare a property use

```
// forward declaration
template<class TypeTag, class MyTypeTag>
struct NewPropTagName;
```

Defining Properties

The value of a property on a given node of the type tag hierarchy is defined using

```
template<class TypeTag>
struct PropertyTagName<TypeTag, TTag::TypeTagName>
{
  // arbitrary body of a struct
};
```

This means a property is defined for a specific type tag node `TTag::TypeTagName` by providing a partial template specialization of `PropertyTagName`. The body typically contains either the alias `type`, or a data member `value`. However, you can of course write in the body whatever you like.

```
template<class TypeTag>
struct PropertyTagName<TypeTag, TTag::TypeTagName> { using type = type; };

template<class TypeTag>
struct PropertyTagName<TypeTag, TTag::TypeTagName> { static constexpr bool value = booleanValue; };

template<class TypeTag>
struct PropertyTagName<TypeTag, TTag::TypeTagName> { static constexpr int value = integerValue; };
```

Example:

```
namespace Dumux {
namespace Properties {

// Create new type tag
namespace TTag {
struct MyTypeTag {};
}

// Define some properties
template<class TypeTag, class MyTypeTag> struct MyCustomProperty { using type = UndefinedProperty; };
```

```
template<class TypeTag, class MyTypeTag> struct MyType { using type = UndefinedProperty; };
template<class TypeTag, class MyTypeTag> struct MyBoolValue { using type = UndefinedProperty; };
template<class TypeTag, class MyTypeTag> struct MyIntValue { using type = UndefinedProperty; };
template<class TypeTag, class MyTypeTag> struct MyScalarValue { using type = UndefinedProperty; };

// Set the properties for the new type tag
template<class TypeTag>
struct MyCustomProperty<TypeTag, TTag::MyTypeTag>
{
    static void print()
    { std::cout << "Hello, World!\n"; }
};

template<class TypeTag>
struct MyType<TypeTag, TTag::MyTypeTag> { using type = unsigned int; };

template<class TypeTag>
struct MyBoolValue<TypeTag, TTag::MyTypeTag> { static constexpr bool value = true; };

template<class TypeTag>
struct MyIntValue<TypeTag, TTag::MyTypeTag> { static constexpr int value = 12345; };

template<class TypeTag>
struct MyScalarValue<TypeTag, TTag::MyTypeTag> { static constexpr double value = 12345.67890; };
}}
```

Retrieving Property Values

The type of a property can be retrieved using

```
using Prop = GetProp<TypeTag, Properties::PropertyTag>;
```

There is a helper struct and a helper function to retrieve the `type` and `value` members of a property

```
using PropType = GetPropType<TypeTag, Properties::PropertyTag>;
constexpr auto propValue = getPropValue<TypeTag, Properties::PropertyTag>();
```

Example:

```
template <TypeTag>
class MyClass {
    // retrieve the ::value attribute of the 'UseMoles' property
    static constexpr bool useMoles = getPropValue<TypeTag, Properties::UseMoles>();
    static constexpr bool useMoles2 = GetProp<TypeTag, Properties::UseMoles>::value; // equivalent

    // retrieve the ::type attribute of the 'Scalar' property
    using Scalar = GetPropType<TypeTag, Properties::Scalar>;
    using Scalar2 = GetProp<TypeTag, Properties::Scalar>::type; // equivalent
};
```

Nesting Property Definitions

Inside property definitions there is access to all other properties which are defined somewhere on the type tag hierarchy. The node for which the current property is requested is available via the template argument `TypeTag`. Inside property class bodies `GetPropType` can be used to retrieve other properties and create aliases.

Example:

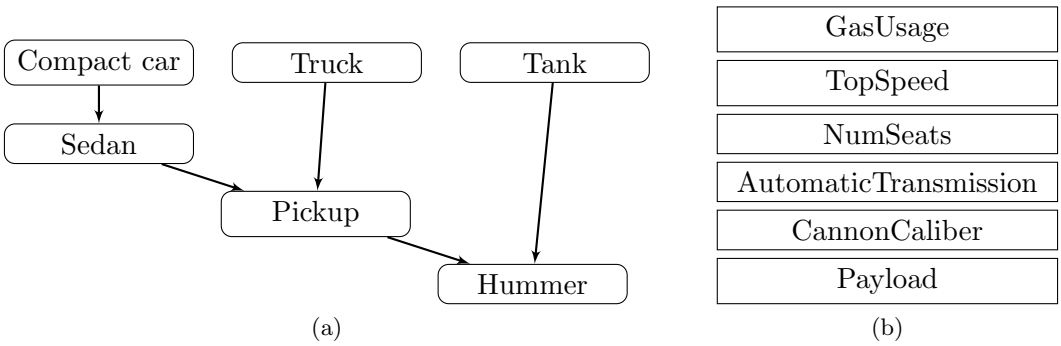


Figure 6.9: **(a)** A possible property inheritance graph for various kinds of cars. The lower nodes inherit from higher ones; Inherited properties from nodes on the right take precedence over the properties defined on the left. **(b)** Property names which make sense for at least one of the car types of (a).

```
template<class TypeTag>
struct Vector<TypeTag, TTag::MyModelTypeTag>
{
    using Scalar = GetPropType<TypeTag, Properties::Scalar>;
    using type = std::vector<Scalar>;
};
```

6.5.3 A Self-Contained Example

As a concrete example, let us consider some kinds of cars: Compact cars, sedans, trucks, pickups, military tanks and the Hummer-H1 sports utility vehicle. Since all these cars share some characteristics, it makes sense to inherit those from the closest matching car type and only specify the properties which are different. Thus, an inheritance diagram for the car types above might look like outlined in Figure 6.9a.

Using the DuMu^x property system, this inheritance hierarchy is defined by:

```
#include <dumux/common/propertysystem.hh>
#include <iostream>

namespace Dumux {
namespace Properties {
namespace TTag{
struct CompactCar {};
struct Truck {};
struct Tank {};

struct Sedan { using InheritsFrom = std::tuple<CompactCar>; };
struct Pickup { using InheritsFrom = std::tuple<Truck, Sedan>; };
struct HummerH1 { using InheritsFrom = std::tuple<Tank, Pickup>; };
}}} // end namespace TTag
```

Figure 6.9b lists a few property names which make sense for at least one of the nodes of Figure 6.9a. These property names can be defined as follows:

```
template<class TypeTag, class MyTypeTag>
struct GasUsage { using type = UndefinedProperty; }; // [l/100km]
template<class TypeTag, class MyTypeTag>
```

```
struct TopSpeed { using type = UndefinedProperty; }; // [km/h]
template<class TypeTag, class MyTypeTag>
struct NumSeats { using type = UndefinedProperty; }; // []
template<class TypeTag, class MyTypeTag>
struct AutomaticTransmission { using type = UndefinedProperty; }; // true/false
template<class TypeTag, class MyTypeTag>
struct CannonCaliber { using type = UndefinedProperty; }; // [mm]
template<class TypeTag, class MyTypeTag>
struct Payload { using type = UndefinedProperty; }; // [t]
```

So far, the inheritance hierarchy and the property names are completely separate. What is missing is setting some values for the property names on specific nodes of the inheritance hierarchy. Let us assume the following:

- For a compact car, the top speed is the gas usage in $^{\mathrm{l}}/_{100\mathrm{km}}$ times 30, the number of seats is 5 and the gas usage is $4\,^{\mathrm{l}}/_{100\mathrm{km}}$.
- A truck is by law limited to $100\,^{\mathrm{km}}/_{\mathrm{h}}$ top speed, the number of seats is 2, it uses $18\,^{\mathrm{l}}/_{100\mathrm{km}}$ and has a cargo payload of 35 t.
- A tank exhibits a top speed of $60\,^{\mathrm{km}}/_{\mathrm{h}}$, uses $65\,^{\mathrm{l}}/_{100\mathrm{km}}$ and features a 120 mm diameter cannon
- A sedan has a gas usage of $7\,^{\mathrm{l}}/_{100\mathrm{km}}$, as well as an automatic transmission, in every other aspect it is like a compact car.
- A pick-up truck has a top speed of $120\,^{\mathrm{km}}/_{\mathrm{h}}$ and a payload of 5 t. In every other aspect it is like a sedan or a truck but if in doubt, it is more like a truck.
- The Hummer-H1 SUV exhibits the same top speed as a pick-up truck. In all other aspects it is similar to a pickup and a tank, but, if in doubt, more like a tank.

Using the DuMu^x property system, these assumptions are formulated using

```
template<class TypeTag>
struct TopSpeed<TypeTag, TTag::CompactCar>
{static constexpr int value = getPropValue<TypeTag, Properties::GasUsage>() * 30};

template<class TypeTag>
struct NumSeats<TypeTag, TTag::CompactCar> { static constexpr int value = 5; };

template<class TypeTag>
struct GasUsage<TypeTag, TTag::CompactCar> { static constexpr int value = 4; };

template<class TypeTag>
struct TopSpeed<TypeTag, TTag::Truck> { static constexpr int value = 100; };

template<class TypeTag>
struct NumSeats<TypeTag, TTag::Truck> { static constexpr int value = 2; };

template<class TypeTag>
struct GasUsage<TypeTag, TTag::Truck> { static constexpr int value = 18; };

template<class TypeTag>
struct Payload<TypeTag, TTag::Truck> { static constexpr int value = 35; };

template<class TypeTag>
struct TopSpeed<TypeTag, TTag::Tank> { static constexpr int value = 60; };

template<class TypeTag>
```

```
struct GasUsage<TypeTag, TTag::Tank> { static constexpr int value = 65; };

template<class TypeTag>
struct CannonCaliber<TypeTag, TTag::Tank> { static constexpr int value = 120; };

template<class TypeTag>
struct GasUsage<TypeTag, TTag::Sedan> { static constexpr int value = 7; };

template<class TypeTag>
struct AutomaticTransmission<TypeTag, TTag::Sedan> { static constexpr bool value = true; };

template<class TypeTag>
struct TopSpeed<TypeTag, TTag::Pickup> { static constexpr int value = 120; };

template<class TypeTag>
struct Payload<TypeTag, TTag::Pickup> { static constexpr int value = 5; };

template<class TypeTag>
struct TopSpeed<TypeTag, TTag::HummerH1>
{ static constexpr int value = getPropValue<TypeTag, TTag::Pickup::TopSpeed<TypeTag>>(); };
```

Now property values can be retrieved and some diagnostic messages can be generated. For example

```
int main()
{
    std::cout << "top speed of sedan: " << getPropValue<Properties::TTag::Sedan, Properties::TopSpeed>() << "\n";
    std::cout << "top speed of truck: " << getPropValue<Properties::TTag::Truck, Properties::TopSpeed>() << "\n";
}
```

will yield the following output:

```
$ top speed of sedan: 210
$ top speed of truck: 100
```

6.6 Input and Output

This section briefly explains grid generation in DuMu^x, summarizes the grid formats that can be used by DuMu^x and introduces the DuMu^x `GridManager`. Finally, this section informs about handling output in DuMu^x.

6.6.1 Supported grid file formats

DuMu^x can read grids from file using the Dune Grid Format (DGF), the Gmsh mesh format (MSH), the Eclipse grid format (GRDECL), or the Visualization ToolKit (VTK/VTU/VTP) format.

Dune Grid Format

Most of our DuMu^x tests use the Dune Grid Format (DGF) to read in grids. A detailed description of the DGF format and some examples can be found in the DUNE doxygen documentation (**Modules → I/O → Dune Grid Format (DGF)**). To generate larger or more complex DGF files, we recommend to write your own scripts, e.g, in C++, Matlab or Python.

The DGF format can also be used to read in spatial parameters defined on the grid. These parameters can be defined on nodes as well as on the elements. An example for predefined parameters on a grid can be found in `dumux/test/porousmediumflow/co2/`.

Gmsh Mesh Format

Gmsh is an open-source flexible grid generator for unstructured finite-element meshes ([11], `http://geuz.org/gmsh/`). DuMu^x supports the default Gmsh mesh format (MSH). For the format specifics and how to create grids with Gmsh, e.g., using the provided GUI, we refer to the Gmsh documentation (`http://geuz.org/gmsh/doc/texinfo/gmsh.html`).

The MSH format can contain element and boundary markers defined on the grid. Thus, boundaries can be easily marked as, e.g., inflow boundaries using Gmsh. Further, the format supports higher order elements. They can be used to create boundary parametrization supported by, e.g., the grid manager `UGGrid`. An example can be found in `dumux/test/io/gridmanager`.

Eclipse Grid Format

The Eclipse Grid Format (GRDECL) is commonly used for corner-point grids. Such grids consist of hexahedra, which are described by eight points on so-called pillars. A special feature of corner-point geometries is that points on pillars can degenerate, meaning that two neighboring points on a pillar can coincide. Furthermore, faces are, in general, bi-linear and cells can be non-convex. This allows for the accurate description of faults, layers, or wells, occurring in geological environments.

Furthermore, petrophysical properties can be defined (for each cell), by using eclipse-specific keywords, e.g. `PORO`, `PERMX`, `PERMY`.

DuMu^x supports the Eclipse Grid Format by using the `opm-grid` module (see (`https://opm-project.org`). An example can be found in `dumux/test/porousmediumflow/2p/cornerpoint`.

VTK File Format

VTK format uses ASCII or XML format. It is mostly used by DuMu^x for output purposes and can be visualized by programs such as Paraview, ViIt or Tecplot. Using VTK files to input grid and parameter data is also possible. An example can be found in `dumux/test/io/gridmanager`.

Other Grid Formats

Grid formats other than DGF, MSH, GRDECL, or VTK will have to be converted to the DGF, MSH, GRDECL, or VTK format before they can be used in DuMu^x. If conversion is not an option, another possibility would be to write your own `GridManager`s. Examples of other grid formats, which have previously been either converted or custom-created in DuMu^x, are ArtMesh grids (fractured network grids), and ICEM grids (CAD developed grids).

6.6.2 The DuMu^x `GridManager`

The `Dumux::GridManager` class constructs the grid from information in the input file and handles the data. Currently, supported Dune grid interface implementations are `YaspGrid`, `OneDGrid`, `dune-uggrid`, `dune-alugrid`, `dune-foamgrid`, `dune-subgrid`, `opm-grid` (cornerpoint grids) and `dune-spgrid`. Grids can be constructed from a DGF, VTK or MSH file by simply providing the filename to the grid in the `Grid` group [2] of the input file:

[2] Note, that group name `Grid` is the default group name and can be customized in your problem changing the string property `GridParameterGroup`. This way, it is possible, e.g., for problems with more than one grid, to set different

```
[Grid]
File = mydgfgrid.dgf
```

If you are using an unstructured grid interface like `UGGrid` or `FOAMGrid`, constructing a grid from a VTK or MSH is just changing a line:

```
[Grid]
File = mygmshgrid.msh
```

DuMu^x will tell you in case your selected grid manager does not support reading such files.

You want to initially refine your grid? It's just adding a line:

```
[Grid]
File = mydgfgrid.dgf
Refinement = 4
```

When reading a MSH or VTK file, further parameters are recognized. `Verbose` enables verbose output on grid construction when set to 1. `BoundarySegments` enables reading parameterized boundaries. `PhysicalEntities` enables reading boundary and element flags.

Parameters specific to the grid implementation

The `Dumux::GridManager` supports also a selection of parameters that are specific to the chosen grid manager. To give an example, we take a look at the unstructured grid `UGGrid`. `UGGrid` supports red-green refinement per default. One can turn off the green closure by setting the grid's closure type

```
[Grid]
File = mydgfgrid.dgf
ClosureType = None # or Green
```

For all available parameters see the Doxygen documentation.

Structured grids

If you want to construct a structured grid without using a specific grid file, insert the following into the input file:

```
[Grid]
LowerLeft = 00 0
UpperRight = 11 1
Cells = 1010 20
```

where `LowerLeft` is a vector to the lower left corner of the grid and `UpperRight` a vector to the upper right corner. `Cells` is a vector with the number of cells in each coordinate direction. Note, that for a grid in a two-dimensional world, the vectors only have two entries.

Depending on the grid manager, further parameters are recognized. `UGGrid`, for example, supports simplex elements as well as hexahedral elements (called "cube" in DUNE). When creating a structured grid, we can select the cell type as follows

```
[Grid]
LowerLeft = 00 0
UpperRight = 11 1
Cells = 1010 20
CellType = Cube # or Simplex
```

For all available parameters see the Doxygen documentation.

group names for each grid, thus configuring them separately.

Other DuMu^x `GridManagers`

- `CakeGridManager`: Provides a method to create a piece of cake grid.
- `CpGridManager`: Reads the GRDECL file and generates a corner-point grid.
- `SubgridGridManager`: Creates a `dune-subgrid` for some given host grid.

6.6.3 Input and Output formats

VTK file format

Dumux allows to write out simulation results via the `VtkOutputModule`. For every print-out step, a single VTU file is created. For parallel simulations one file per print-out step is generated for each processor. The PVD file groups the single VTU files and contains additionally the time step information. The VTK file format is supported by common visualisation programs like ParaView, VisIt, and Tecplot.

Customize the VTK output

Using the respective `initOutputModule` function of the model `IOFields`, a default set of variables is defined for the output into the VTK files. It is also possible to add further variables, using the member function `addField` of the `VtkOutputModule`. For example, to add a variable called `temperatureExact`:

```
vtkWriter.addField(problem->getExactTemperature(), "temperatureExact");
```

The first input argument of this method is the value of the additional variable, provided by a method of the corresponding problem. If it does not already exists, the user has to provide this method.

```
//! get the analytical temperature
const std::vector<Scalar>& getExactTemperature()
{
    return temperatureExact_;
}
```

It is important that the life-time of the added field exceeds the life-time of the writer. That means you can't pass temporaries to the `addField` function. The vector has to be stored somewhere, e.g. in the program main file.

The second input argument is the name of the additional variable (as it should be written in the VTK files). The example above is taken from:
`test/porousmediumflow/1pnc/implicit/1p2c/nonisothermal/convection/main.cc`

VTK as input format

There is support for reading data and grids from VTK files, see subsection 6.6.1.

Gnuplot interface

DuMu^x provides a small interface to GNUPlot, which can be used to plot results and generate image files (e.g., png). To use the gnuplot, gnuplot has to be installed. For more information see 5.6.2.

Container I/O

DuMu^x supports writing to file from and reading to some standard C++ containers like `std::vector<double>` or `std::vector<Dune::FieldVector>`. If you want to read and write simple vectors, have a look at the header `dumux/io/container.hh`.

Matrix and Vector I/O

`dune-istl` supports writing and reading vectors and matrices to/from different format. For example you can write a matrix in a sparse matrix format that can be read by Matlab (see `dune/istl/io.hh`).

6.7 Parallel Computation

This section explains how DuMu^x can be used on multicore/multinode systems.

There are different concepts and methods for parallel programming, which are often grouped in *shared-memory* and *distributed-memory* approaches. The parallelization in DuMu^x is based on the model supported by Dune which is currently based on *Message Passing Interface* (MPI) (distributed-memory approach).

The main idea behind the MPI parallelization is the concept of *domain decomposition*. For parallel simulations, the computational domain is split into subdomains and one process (*rank*) is used to solve the local problem of each subdomain. During the global solution process, some data exchange between the ranks/subdomains is needed. MPI is used to send data to other ranks and to receive data from other ranks. The domain decomposition in Dune is handled by the grid managers. The grid is partitioned and distributed on several nodes. Most grid managers contain own domain decomposition methods to split the computational domain into subdomains. Some grid managers also support external tools like METIS, ParMETIS, PTScotch or ZOLTAN for partitioning. On the other hand, linear algebra types such as matrices and vectors do not know that they are in a parallel environment. Communication is then handled by the components of the parallel solvers. Currently, there are two parallel solver backends available. First, the `Dumux::AMGBiCGSTABBackend`, a parallel AMG-preconditioned BiCGSTAB solver. Second, the `Dumux::IstlSolverFactoryBackend`, which provides a selections of different parallel solvers and preconditioners. This backend makes it also possible to choose solver and preconditioner during runtime, but this flexibility is achieved by the cost of an increased compile time.

In order for DuMu^x simulation to run in parallel, an MPI library (e.g. OpenMPI, MPICH or IntelMPI) implementation must be installed on the system. However, not all parts of DuMu^x can be used in parallel.

Furthermore, we note that the parallel AMG preconditioner of `dune-istl` defaults to an iterative SSOR coarse grid solver if no direct solver is found on your system. Unfortunately, the iterative solver has a very high and hard-coded tolerance as a termination criterion, which will not solve the coarse grid system with sufficient accuracy for typical problems in DuMu^x. We therefore recommend to install one of the direct solver libraries supported by `dune-istl`. This is either UMFPack contained in SuiteSparse, or SuperLU, see Section 3.5.1.

6.7.1 Prepare a Parallel Application

In order to switch to a parallel solver backend include the respective header

```
#include <dumux/linear/amgbackend.hh>
```

or

```
#include <dumux/linear/istlsolverfactorybackend.hh>
```

Second, the linear solver must be switched to the parallel solver backend

```
using LinearSolver = AMGBiCGSTABBackend<LinearSolverTraits<GridGeometry>>;
```

or

```
using LinearSolver = IstlSolverFactoryBackend<LinearSolverTraits<GridGeometry>>;
```

The parallel instance of the linear solver has to be constructed with a `Dune::GridView` object and a mapper, in order to construct the parallel index set needed for communication.

```
auto linearSolver = std::make_shared<LinearSolver>(leafGridView, gridGeometry->dofMapper());
```

When using the `Dumux::IstlSolverFactoryBackend`, solver and preconditioner have to be specified in the file `params.input` by the parameters `LinearSolver.Type` and `LinearSolver.Preconditioner.Type`. Possible solvers are `bicgstabsolver` or `restartedgmressolver`, possible preconditioners `ilu` or `gs`. Depending on the chosen solver and preconditioner additional parameters can be specified. If not the global default parameters are used as shown in section 5.3. More information about the options and the other available solvers and preconditioners can be found in `dune-istl` (`solvers.hh` and `preconditioners.hh`).

6.7.2 Run a Parallel Application

The starting procedure for parallel simulations depends on the chosen MPI library. Most MPI implementations use the **mpirun** command

```
mpirun -np <n_cores> <executable_name>
```

where *-np* sets the number of cores (`n_cores`) that should be used for the computation. On a cluster you usually have to use a queuing system (e.g. slurm) to submit a job. Check with your cluster administrator how to run parallel applications on the cluster.

6.7.3 Handling Parallel Results

For serial computations, DuMu^x produces single vtu-files as default output format. During a simulation, one VTU file is written for every output step. In the parallel case, one VTU file for each step and processor is created. For parallel computations, an additional variable `"process rank"` is written into the file. The process rank allows the user to inspect the subdomains after the computation. The parallel VTU files are combined in a single pvd file like in sequential simulations that can be opened with e.g. ParaView.

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